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The Art of Granular Computing

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Abstract. The current research in granular computing is dominated by set-theoretic models such as rough sets and fuzzy sets. By recasting the existing studies in a wider context, we propose a unified framework of granular computing. The new framework extends results obtained in the set-theoretic setting and extracts high-level common principles from a wide range of scientific disciplines. The art of granular computing for problem solving emerges from the resulting common philosophy, methodology and information processing paradigm. Granular computing stresses not only the need for rigor, structure, conciseness and clarity, but also the importance of conscious effects and wisdom in using powerful strategies and heuristics in stating and solving problems.

Keywords. Granular computing triangle, structured thinking, structured problem solving, structured information processing

1 Introduction

The advances of rough set theory have greatly influenced the development of granular computing [4, 5, 19, 33, 34, 36, 43, 46, 47, 51, 52, 62, 65, 66]. Specifically, the philosophy and methodology of rough sets, centralized on the notions of indiscernibility and knowledge granularity, are fundamental to granular computing. It is fair to say that the plentiful results and applications of the theory of rough sets motivate many researchers to study granular computing.

An underlying notion of rough set theory is an equivalence relation representing indiscernibility of objects and the induced partition of a universe [45–47]. Suppose a finite universe of objects is described by a finite set of attributes in the form of an information table. Different equivalence relations can be constructed based on distinct subsets of attributes [45]. One may interpret a partition as a simple flat granulated view of the universe with each equivalence class as a granule. Under this view, rough set analysis deals with approximation and reasoning with partitions of different levels of granularity [45, 66].

The basic ideas of partitioning a universe for problem solving have also been used in many studies such as the partition model of databases [31], the theory of granularity [21], and the quotient space theory [74, 75]. Each of these studies is formulated differently to deal with a different type of problems. In spite of their differences, they all share two common features with the rough set theory. First, they all consider different descriptions of the same problem at multiple levels of

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granularity. This allows us to focus on solving a problem at the most appropriate level of granularity by ignoring unimportant and irrelevant details. The second feature is that multilevel descriptions are linked together to form a hierarchical structure. In other words, levels with differing granularity are partially ordered. This allows us to change granularity easily at different stages of problem solving.

These two features are common to problem solving activities across many branches of science [68–70]. Although scientists in different disciplines study different subject matters and use different formulations, they all employ remarkably common structures for describing problems and apply common principles, strategies, and heuristics for problem solving [7, 39]. Our understanding and formulation of granular computing is based on such high-level features [66–69]. We attempt to extract the common domain-independent principles, strategies and heuristics that have been applied either explicitly or implicitly in many disciplines. The results enable us to arrive at a unified framework of granular computing for problem solving from three perspectives [69].

The main objective of this paper is to explore granular computing as the creative art of problem solving. We propose and examine a trinity framework of granular computing.

2 Granular Computing as a New Field of Study

In the past ten years, many researchers have focused their efforts on the development of a new research field under the umbrella name of granular computing [4, 23, 35, 43, 46, 48, 51, 52, 63, 64, 72, 73]. Extensive results and applications demonstrate the need for and the potential of granular computing. However, the advance of granular computing suffers from the lack of a conceptual framework that enables us to answer some of the fundamental questions. In order to justify the existence of granular computing as a new field in its own right, we need to address these questions.

It is a well-accepted fact that the basic ideas, principles and strategies of granular computing appear in many branches of science and different fields of computer science [4, 70, 72]. This immediately raises the following questions:

- What is new and unique in granular computing?
- What are the contributions of granular computing?
- What are the relations between granular computing and other fields?
- What are the scopes and goals for the study of granular computing?

The answers to these questions show the necessity for the study of granular computing, provide the context to which granular computing fits, and set the goals of research on granular computing.

People solve different problems by using some common principles. However, one can make several important observations regarding their actual usages. First, these principles are scattered over many places in isolation without being synthesized into an integrated whole. Second, they are normally explained with reference to domain-specific knowledge and thus are buried deeply in minute details.

Third, the same principles are discussed in different languages and notations. Fourth, these principles are typically used either implicitly or subconsciously, for a formal documentation does not exist. They are not readily accessible for many people to use. Sometimes, the same principles are reinvented time and again in the same or different fields. By introducing granular computing as a new field of study, we attempt to resolve such problems.

To a large extent, the emergence of granular computing is motivated by the same reasons that led to the introduction of general systems theory several decades ago [9, 29, 44, 56]. As a new field of research, granular computing is a study of the art of problem solving. It has two unique tasks. One is to extract high-level commonalities of different disciplines and to synthesize their results into an integrated whole by ignoring low-level details. The other is to make explicit ideas hidden in discipline-specific discussions in order to arrive at a set of discipline-independent principles.

What makes granular computing new and unique is not its individual principles, methodologies, and strategies. Each of them has been extensively studied by authors in many fields. Granular computing contributes by synthesizing, integrating, and studying them in a uniform way. Through granular computing, we attempt to achieve the following goals:

- to make implicit principles explicit,
- to make invisible principles visible,
- to make domain-specific principles domain-independent,
- to make subconscious effects conscious.

It is possible to empower more people with effective strategies for problem solving tasks. One can consciously apply the principles of granular computing in solving a wide range of problems. It is also possible to prevent a waste of research efforts rediscovering or reinventing these principles.

Granular computing is a multidisciplinary study that emerged from existing disciplines and fields of study. For example, in addition to rough sets [45–47] and fuzzy sets [72, 73], granular computing can draw results from the following:

- philosophy and philosophy of science [37, 49],
- research methods [7, 39],
- cognitive science and cognitive psychology [53, 58],
- human problem solving [42],
- general systems theory [9, 29, 56],
- synectics [18],
- hierarchy theory [1, 44, 55, 56, 60],
- cluster analysis [2],
- social networks [3, 24],
- artificial intelligence [16, 20, 21, 27, 74],
- learning [11, 50, 57],
- computer programming [12, 28, 30, 61],
- information processing [4, 26, 38],
- teaching and instruction [13, 54],

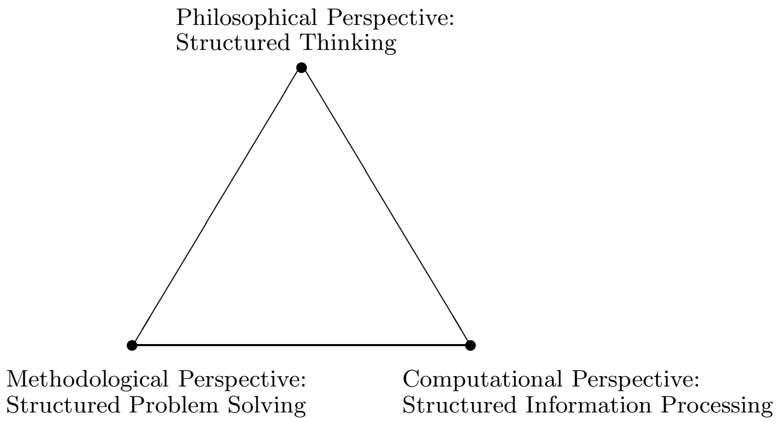

Fig. 1. The granular computing triangle

- rhetoric and writing [14, 22, 41, 71].

This list is not intended to be exhaustive, but an illustration to show the diversity of disciplines where the principles of granular computing can be observed.

A theory of granular computing may be established by extracting, sorting, integrating, synthesizing, and interpreting a set of generally applicable principles, methods, and strategies for problem solving. In the past few years, many researchers have made significant progress on concrete models and methods of granular computing. In the meantime, one can also observe a number of studies that simply restate existing results using the terminology of granular computing or reinvent them in a different context. A conceptual study of granular computing may free us from similar pitfalls.

3 The Granular Computing Triangle

Granular computing can be studied from three perspectives that are unified and based on the notion of granular structures. The granular computing triangle of Figure 1 represents this trinity view. In the philosophical perspective, granular computing deals with structured thinking. It attempts to extract and formalize human thinking. In the methodological perspective, granular computing concerns structured problem solving. It aims to study methods and techniques for systematic problem solving. In the computational perspective, granular computing is a paradigm of structured information processing. It addresses the problems of information processing in the abstract, in the brain, and in machines. Each perspective supports the other two perspectives. What integrates them is the granular structures that represent the real world at multiple levels of granularity. By emphasizing on structures, granular computing leads to structured solutions to real-world problems.

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3.1 Granular Structures

A primitive notion of granular computing is a granule representing a part of a whole. Like systems theory, granular computing explores the composition of parts, their interrelationships, and connections to the whole. A real-world problem normally consists of a web of interacting and interrelated parts [9]. In order to have a practical understanding and solution, it is necessary to extract approximate structures that are tractable and easy to analyze. Granular computing exploits structures in terms of granules, levels, and hierarchies based on multilevel and multiview representations [69].

A granule plays two distinctive roles. It may be an element of another granule and is considered to be a part forming the other granule. It may also consist of a family of granules and is considered to be a whole. Its particular role is determined by our focal points at different stages of problem solving. This part-whole relationship suggests a partial ordering of granules. It is possible to derive a hierarchical structure. The term hierarchy is used to denote such a structure that consists of a family of interacting and interrelated granules, and each of them can be, in turn, a hierarchical structure. Trees and lattices are typical examples of hierarchical structures. Another example is the notion of rule complex, introduced and elaborated by Burns and Gomolińska [8, 17] within the generalized game theory. We may view a hierarchy as a structure of (partially) ordered multiple levels. Each level is made up of a family of granules. Hierarchical structures not only make a complex problem more easily understandable, but also lead to efficient, although perhaps approximate, solutions.

In building a hierarchical structure, we need to have a vertical separation of levels and a horizontal separation of granules at the same hierarchical level. These separations explore the notion of approximations and a loose coupling of parts [9, 56]. In forming a granule, one may ignore the subtle differences between its elements as well as their individual connections to others. That is, a group of elements may be treated approximately as a whole when studying their relations to others. Each level may be viewed as a representation of a problem at a specific level of granularity. The relationship between levels can be interpreted in terms of abstraction, control, complexity, detail, resolution, etc.

A hierarchy represents the results of a study of a problem from one particular angle or point-of-view. Some useful information may be lost with a hierarchy instead of a web. For the same problem, many interpretations and descriptions may co-exist [6, 10]. It may be necessary to construct and compare multiple hierarchies [24]. A comparative study of those hierarchies may provide a complete understanding of the problem.

In summary, granular computing exploits multilevel and multiview representations in problem solving. A hierarchy represents one view of a problem with multiple levels of granularity. Depending on different contexts of applications, we may have data granulation, information granulation, and knowledge granulation corresponding to granular data structures, granular information structures, and granular knowledge structures.

3.2 Structured Thinking

Granular computing, as structured thinking, integrates two complementary philosophical views dealing with the complexity of real-world problems, namely, the traditional reductionist thinking and the more recent systems thinking. It stresses the importance of conscious effects in thinking with hierarchical structures.

According to reductionist thinking, a complex system or problem can be divided into simpler and more fundamental parts, and each part can be further divided. An understanding of the system can be reduced to the understanding of its parts. In other words, we can deduce fully the properties of the system based solely on the properties of its parts. In contrast, systems thinking shifts from parts to the whole, in terms of connectedness, relationships, and context [9, 29]. A complex system is viewed as an integrated whole consisting of a web of interconnected, interacting, and highly organized parts. The properties of the whole are not present in any of its parts, but emerge from the interactions and relationships of the parts.

The reductionist thinking and systems thinking agree on the modeling of a complex system in terms of the whole and parts, but differ in how to make inference with the parts. Based on this common hierarchical structure, granular computing attempts to unify reductionist thinking and systems thinking into structured thinking.

3.3 Structured Problem Solving

Structured thinking leads to a perception and understanding of a real-world problem in terms of multilevel and multiview representations. These structures play a crucial role in problem solving. Granular computing is structured problem solving guided by structured thinking.

Structured problem solving methods and strategies have been extensively studied by many authors. A convincing way to show the effectiveness of granular computing is to present a set of principles and to demonstrate the working of these principles in real-world applications. We present three such principles:

- the principle of multilevel granularity,
- the principle of focused effort,
- the principle of granularity conversion.

The first principle emphasizes the importance of modeling in terms of hierarchical structures. Once such structures are obtained, the second principle calls for attention on the focal point at a particular stage of problem solving. The third principle links the different stages in this process.

Although principles of granular computing are named differently in different disciplines, they are indeed the same at a more abstract level. We briefly summarize the applications of such ideas and principles in several related areas:

Concept formulation and learning: A concept represents a basic unit of human thought and is commonly labeled by a word of a natural language. Hierarchical structures are commonly used in organizing human knowledge [49, 54]. To a large extent, human learning is a good example to demonstrate the working principles of granular computing, i.e., attention and changing of attention.

Structured programming: Hierarchical structures are a central notion to structured programming [12, 30]. In this context, a granule may be viewed as a program module. The stepwise refinement process explores multilevel development of a full program, from a brief high-level description to the final complete program [61].

Structured proofs: Following the results of structured programming, several authors studied structured methods for developing, teaching and communicating mathematical proofs [15, 32]. In particular, a structured method arranges the proof in levels and proceeds in a top-down manner. A level consists of short autonomous modules, each embodying one major idea of the proof to be further concretized in the subsequent levels. The process continues by supplying more details of the higher levels until a complete proof is reached.

Structured writing: Writing may be viewed as a problem solving process and task [14, 41, 71]. A simple idea is described by a paragraph consisting of several sentences. A point-of-view is jointly described and supported by several ideas. A theme emerges from a few different points-of-view. The units of writing are sometimes referred to as information blocks [22], issues [14], ideas [41], and units of experience [71]. For effective communication, one needs to organize them into a hierarchical structure, referred to as an issue tree [14], a pyramid structure of ideas [41], or a hierarchically structured system of units of experience [71]. Like structured programming, structured writing may be viewed as a stepwise refinement that produces a full article.

Two important features can be observed from these studies. One is the construction of building blocks (i.e., granules) and the other is the arrangement of blocks into a hierarchical structure. The ideas, principles, proverbs, maxims, and strategies from these fields can be easily transferred to each other.

Human concept formation and learning determine, in principle, the ways to produce easily-understandable solutions to a problem. For example, the chunking principle underlying human memory [40] suggests a hierarchical structure used in writing [22, 41]. The hierarchical structures of complex systems [56] are applicable to the process of writing if one considers an article to be a complex system that has evolved through time [71]. On the other hand, the styles of programming [25, 30] are influenced by styles of writing English prose [59]. Structured programming in turn offers solutions to structured mathematical proofs [15, 32]. In summary, these examples provide us convincing evidence that supports the study of granular computing. Instead of reinventing the same principles and strategies, one can focus on their applications across many disciplines.

3.4 Structured Information Processing

In the information processing paradigm [4], granular computing works with a pyramid consisting of different-sized information granules. This structured information processing is a necessary feature of any knowledge-intensive system.

Two notions of structured information processing are representation and process [38]. A representation is a formal system that makes explicit certain entities or types of information and a specification of how the system does it. The result is called a description of the entity in the representation. A process may simply be interpreted as actions or procedures for carrying out information processing tasks. In general, a representation determines the effectiveness of processes.

A representation of granules must capture their essential features and make explicit a particular aspect of their physical meanings. It needs to be closely connected to the representations of granular structures with respect to granules, levels, and hierarchies. Processes of granular computing may be broadly divided into the two classes: granulation and computation with granules [64, 70]. Granulation processes involve the construction of the building blocks and structures, namely, granules, levels, and hierarchies. Computation processes systematically explore the granular structures. This involves two-way communications up and down in a hierarchy, as well as switching between levels.

Structured information processing is a stepwise refinement process. At a higher level, one may produce an approximate, a partial, or a schematic solution. The latter is to be made more precise, complete, and detailed at a lower level. The process stops when a desirable (approximate) solution is obtained.

4 Conclusion

A new understanding of granular computing is presented. Granular computing draws extensive results from existing disciplines and offers its own insights and solutions. Its future depends critically on a right balance between the two. We need to draw results from classical thinking and explore new ways of creative thinking. The proposed trinity framework casts granular computing into a wider context. The art of granular computing can be fully appreciated from the philosophical perspective as structured thinking, from the methodological perspective as structured problem solving, and from the computational perspective as structured information processing.

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