Formulating Three-way Decision Making with Game-theoretic Rough Sets

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Outline

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Three-way Decision Making

- Three-way decisions are common in every day life.
- For example, medical field
  - Doctor examining a patient for a possible disease.
  - A disease may be present or absent.
  - Based on evidence the doctor can decide whether to treat, don't treat or do further examination to reach a conclusion.
- Other examples may be found in spam email filtering, investment decisions and peer review process.
Three-way Decision Making

- The problem of three-way decisions [8]

Considering $U$ as a finite nonempty set of objects and $C$ as a set of criteria. The problem of three-way decisions is to divide $U$ based on $C$ into three disjoint regions, POS, NEG and BND called as positive, negative and boundary regions, respectively.
Rough Sets

- Sets derived from imperfect, imprecise, and incomplete data may not be able to be precisely defined.
- Sets have to be approximated.
- Approximating a concept $C$ with objects in $U$.
  - Lower approximation given by $\text{apr}(C)\{x \in U| x \subseteq C\}$.
  - Upper approximation given by $\overline{\text{apr}}(C)\{x \in U| x \cap C \neq \emptyset\}$.
- The three regions defined by the approximation
  - $\text{POS}(C) = \text{apr}(C)$
  - $\text{BND}(C) = \overline{\text{apr}}(C) - \text{apr}(C)$.
  - $\text{NEG}(C) = U - (\text{POS}(C) \cup \text{BND}(C))$. 
Rough Sets

The Set \( C \)

Lower approximation

Boundary region
Probabilistic Rough Sets

• Defines the approximations in terms of conditional probabilities.

• Introduces a pair of threshold denoted as \((\alpha, \beta)\) to determine the probabilistic rough set approximations given by,

\[
\underset{\text{apr}}{\text{apr}}_{(\alpha, \beta)}(C) = \bigcup \{ [x] \in U/E \mid Pr(C|[x]) \geq \alpha \},
\]

\[
\bar{\text{apr}}_{(\alpha, \beta)}(C) = \bigcup \{ [x] \in U/E \mid Pr(C|[x]) > \beta \}. \quad (1)
\]

• Probabilistic positive, negative and boundary regions:

\[
\text{POS}_{(\alpha, \beta)}(C) = \{ x \in U \mid Pr(C|[x]) \geq \alpha \},
\]

\[
\text{NEG}_{(\alpha, \beta)}(C) = \{ x \in U \mid Pr(C|[x]) \leq \beta \},
\]

\[
\text{BND}_{(\alpha, \beta)}(C) = \{ x \in U \mid \beta < Pr(C|[x]) < \alpha \}. \quad (2)
\]
Three-way Decisions with Probabilistic Rough Sets

• Three-way decisions are made according to the following rules.

Acceptance: if \( P(C|x) \geq \alpha \),
Rejection: if \( P(C|x) \leq \beta \), and
Deferment: if \( \beta < P(C|x) < \alpha \). \hspace{1cm} (3)
Determination of thresholds \((\alpha, \beta)\)

- Many attempts have been made to determine the thresholds.
  - Xiuyi Jia’s [2] optimization viewpoint,
  - Huaxiong Li’s [3] multi-view model,
  - Dun Liu’s [4] method using probabilistic model criteria,
  - Deng and Yao [7] information-theoretic interpretation,
  - Yao and his colleagues [1, 5, 6] game-theoretic framework.
Game Theory

- Game theory is a core subject in decision sciences.
- The basic game components include:
  - Players.
  - Strategies.
  - Payoffs.
- A classical example in Game Theory: The prisoners dilemma.

<table>
<thead>
<tr>
<th>( p_1 )</th>
<th>Confess</th>
<th>( p_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confess</td>
<td>( p_1 ) serves 10 year, ( p_2 ) serves 10 years</td>
<td>( p_1 ) serves 0 year, ( p_2 ) serves 20 years</td>
</tr>
<tr>
<td>Don’t confess</td>
<td>( p_1 ) serves 20 year, ( p_2 ) serves 0 years</td>
<td>( p_1 ) serves 1 year, ( p_2 ) serves 1 years</td>
</tr>
</tbody>
</table>
Game-theoretic Rough Sets

- Utilizing a game-theoretic setting for analyzing rough sets.
- Determining the probabilistic thresholds to obtain the three regions and the implied three-way decisions.
- Examples
  - Game of improving classification ability.
  - Game of for obtaining effective rules.
  - Game for reducing region uncertainties.
Game-theoretic Rough Sets

\[
\begin{array}{cccc}
  & s_1 & s_2 & s_3 & s_4 \\
  s_1 & & & & \\
  s_2 & & & & \\
  s_3 & & & & \\
  s_4 & & & & \\
\end{array}
\]

\[(s_3 \cdot s_3) = (\alpha_3, \beta_3)\]
In probabilistic rough sets the criterion of conditional probability of a concept is used to obtain three-way decisions.

We try to incorporate multiple criteria in making three-way decisions.

The GTRS can play a role here.
Formulation of Three-way Decisions with GTRS

- A key observation in probabilistic rough set model.
  - Three-way decisions are based on evaluation of an equivalence class with respect to a concept.
- We utilize a similar idea but with different evaluations based on multiple criteria.
- A two-player game is considered for each equivalence class.
Formulation of Three-way Decisions with GTRS

- **Game details**
  - **Players:** They reflect different criteria used in evaluating a particular equivalence class. For example, gini index, entropy.
  - **Strategies:** Two strategies are considered.
    - $s_1 = P$, is desired when the evaluation result with a criterion are above certain expectation.
    - $s_2 = N$, is desired when the evaluation result with a criterion is below certain expectation.
  - **Payoff Functions:** Defined in terms of measures that are used to evaluate a criterion.
    - The criterion of uncertainty evaluated with Shannon entropy.
Three-way Decisions with GTRS

- The game for three-way decision making

<table>
<thead>
<tr>
<th></th>
<th>$C_1$</th>
<th>$C_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>$u_{C_1}(P,P)$, $u_{C_2}(P,P)$</td>
<td>$u_{C_1}(P,N)$, $u_{C_2}(P,N)$</td>
</tr>
<tr>
<td>$N$</td>
<td>$u_{C_1}(N,P)$, $u_{C_2}(N,P)$</td>
<td>$u_{C_1}(N,N)$, $u_{C_2}(N,N)$</td>
</tr>
</tbody>
</table>

- The solution concept of Nash equilibrium is used to determine the game outcome.
Three-way Decisions with GTRS

- Three possible outcomes of the game.
  - Both players select.
    \[ u_{C_1}(P, P) \geq u_{C_1}(N, P) \quad \& \quad u_{C_2}(P, P) \geq u_{C_1}(P, N). \] \( (4) \)
  - Both Players reject.
    \[ u_{C_1}(N, N) \geq u_{C_1}(P, N) \quad \& \quad u_{C_2}(N, N) \geq u_{C_1}(N, P) \] \( (5) \)
  - One of the players select.
    \[ u_{C_1}(P, N) \geq u_{C_1}(N, N) \quad \& \quad u_{C_2}(P, N) \geq u_{C_1}(P, P) \] \( (6) \)
    \[ u_{C_1}(N, P) \geq u_{C_1}(P, P) \quad \& \quad u_{C_2}(N, P) \geq u_{C_1}(N, N) \] \( (7) \)
Three-way Decisions with GTRS

- Three-way decisions are made according to the following rules.

Acceptance: if \( u_{C_1}(P, P) \geq u_{C_1}(P, N) \) \& \( u_{C_2}(P, P) \geq u_{C_2}(N, P) \),

Rejection: if \( u_{C_1}(N, N) \geq u_{C_1}(N, P) \) \& \( u_{C_2}(N, N) \geq u_{C_2}(P, N) \),

Deferment: \( otherwise \) (8)
Conclusion

- The probabilistic rough sets uses a single criterion of conditional probability to obtain three-way decisions.
- The game-theoretic rough set model can incorporate multiple criteria to obtain three-way decisions.
- The consideration of multiple criteria may allow for more informed and flexible decisions.
Reference (Partial) I


Reference (Partial) II


Questions?