Actionable Three-way Decisions (A3WDs)

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Outline

- Introduction
  - An illustrative example
  - Motivations and objectives
- An A3WD framework
  - Trisecting
  - Acting
- The R4 reduction framework for A3WDs
- Experimental results
- Conclusions and future works

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An Illustrative Example

- Given: a medical decision table
- Goal: cure people who have certain disease

Table 3.1: A decision table for medicine.

#	sex	chol	bp	result
o_1	female	medium	normal	+
o_2	female	medium	normal	-
o_3	female	low	normal	+
o_4	female	low	normal	-
o_5	female	low	normal	-
o_6	female	medium	low	+
o_7	female	high	high	-
o_8	male	high	low	-
o_9	male	low	normal	+

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An Illustrative Example

- Given: a medical decision table
- Goal: cure people who have certain disease

Table 3.1: A decision table for medicine.

#	sex	chol	bp	result
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o_2	female	medium	normal	-
o_3	female	low	normal	+
o_4	female	low	normal	-
o_5	female	low	normal	-
o_6	female	medium	low	+
o_7	female	high	high	-
o_8	male	high	low	-
o_9	male	low	normal	+

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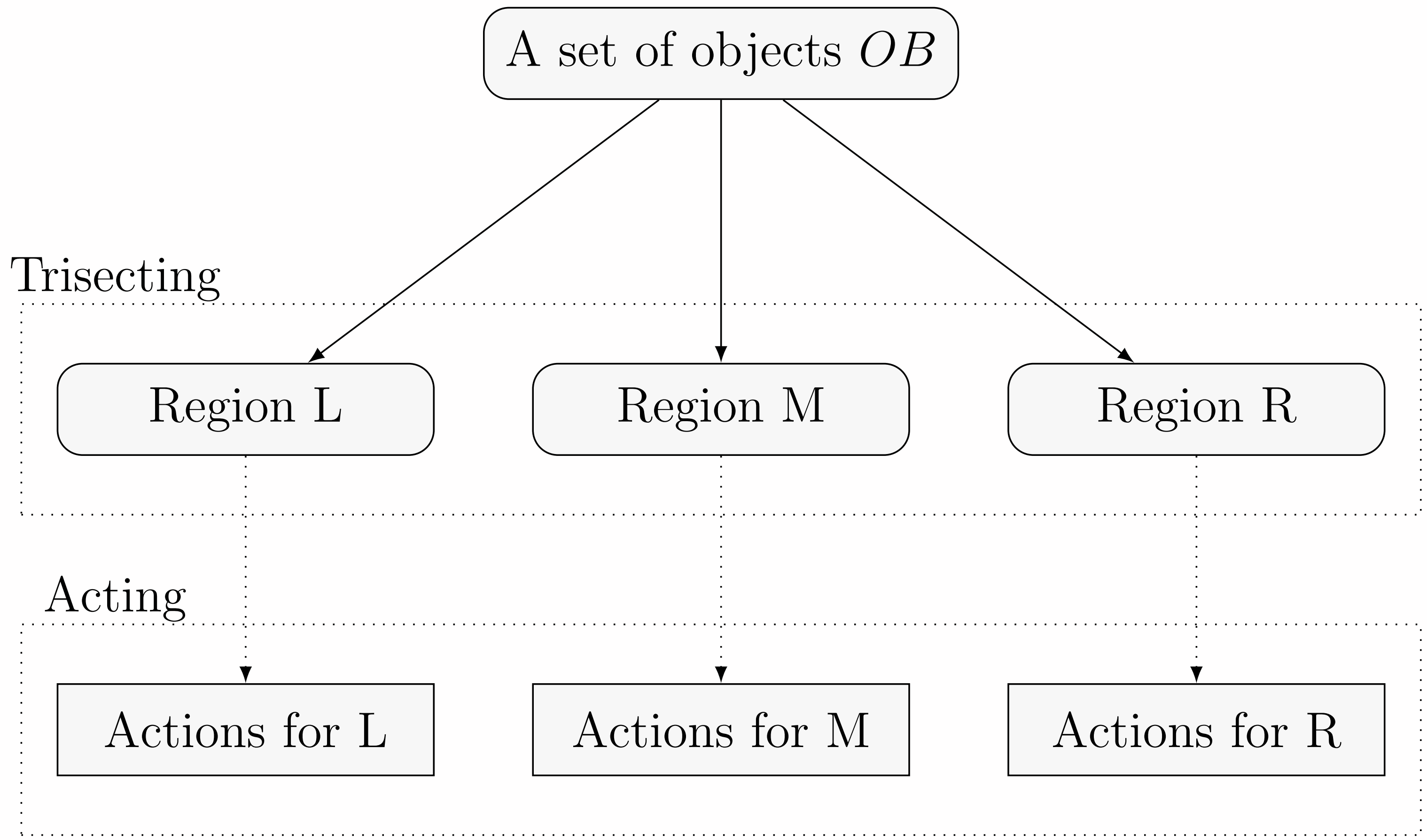
An Illustrative Example (cont.)

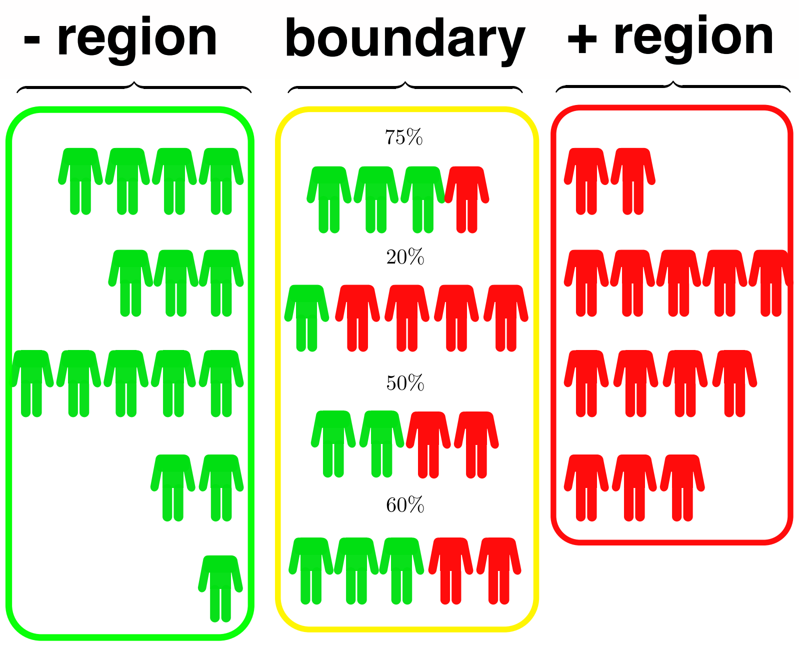
- Three-way decisions (3WDs) [1] can be applied to the problem



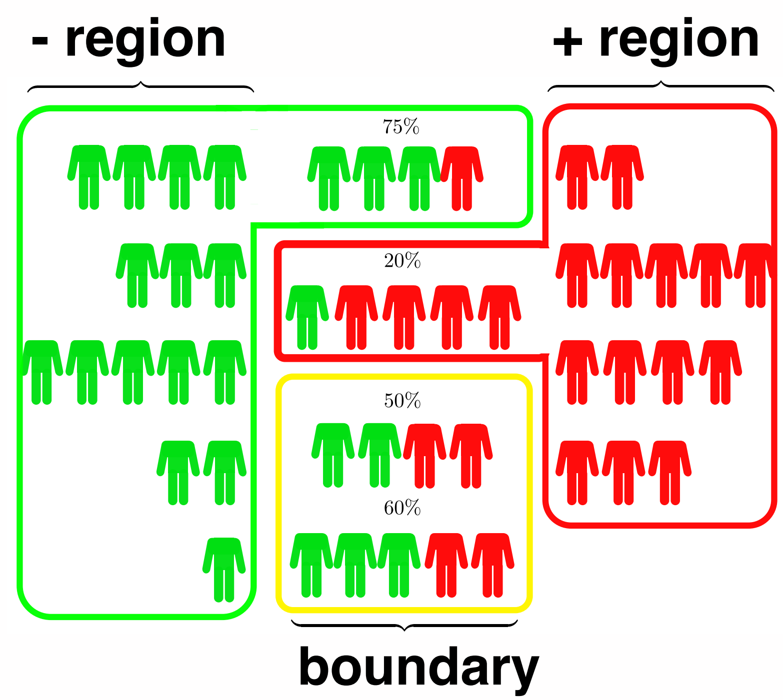
[1] Y.Y. Yao. Three-way decision: an interpretation of rules in rough set theory. In Proceedings of the International Conference on Rough Sets and Knowledge Technology, pp. 642-649, 2009.

An Illustrative Example (cont.)

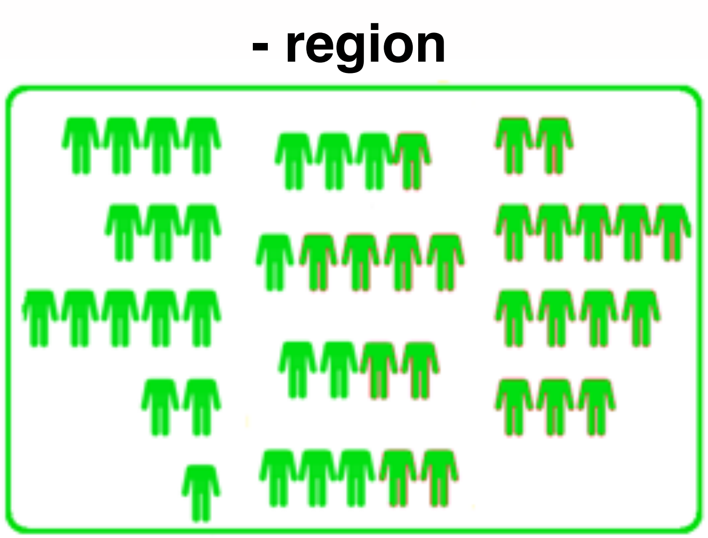
- Trisecting (diagnosis)

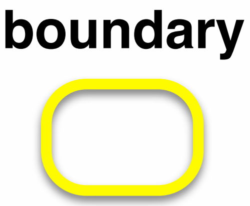


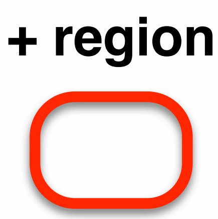
or



- Acting (treatment)







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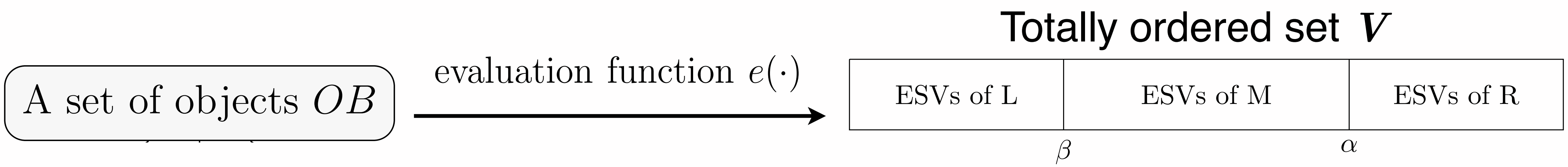
Trisecting

- Trisecting: divide a universal set into three regions



- Three regions:

$$\begin{aligned} \mathrm{L}_{(\alpha,\beta)}(e) &= \{x \in OB \mid e(x) \preceq \beta\}, \\ \mathrm{M}_{((\alpha,\beta)}(e) &= \{x \in OB \mid \beta \prec e(x) \prec \alpha\}, \\ \mathrm{R}_{((\alpha,\beta)}(e) &= \{x \in OB \mid e(x) \succeq \alpha\}. \end{aligned}$$

- Measurement of three regions:

$$Q(\pi) = w_L Q(\mathrm{L}) + w_M Q(\mathrm{M}) + w_R Q(\mathrm{R})$$

- Interpretations of trisecting
  - Cost [2], entropy [3], Gini index [4], and game [5]

[2] Y.Y. Yao. Decision-theoretic rough set models. In Proceedings of the International Conference on Rough Sets and Knowledge Technology, pp. 1-12, 2007.
[3] X.F. Deng, Y.Y. Yao. A multifaceted analysis of probabilistic three-way decisions. Fundamenta Informaticae, 132(3): 291-313, 2014.
[4] Y. Zhang, J.T. Yao. Gini objective functions for three-way classifications. International Journal of Approximate Reasoning, 81: 103-114, 2017.
[5] J.T. Yao, J.P. Herbert. A game-theoretic perspective on rough set analysis. Journal of Chongqing University of Posts and Telecommunications, 20: 291-298, 2008.

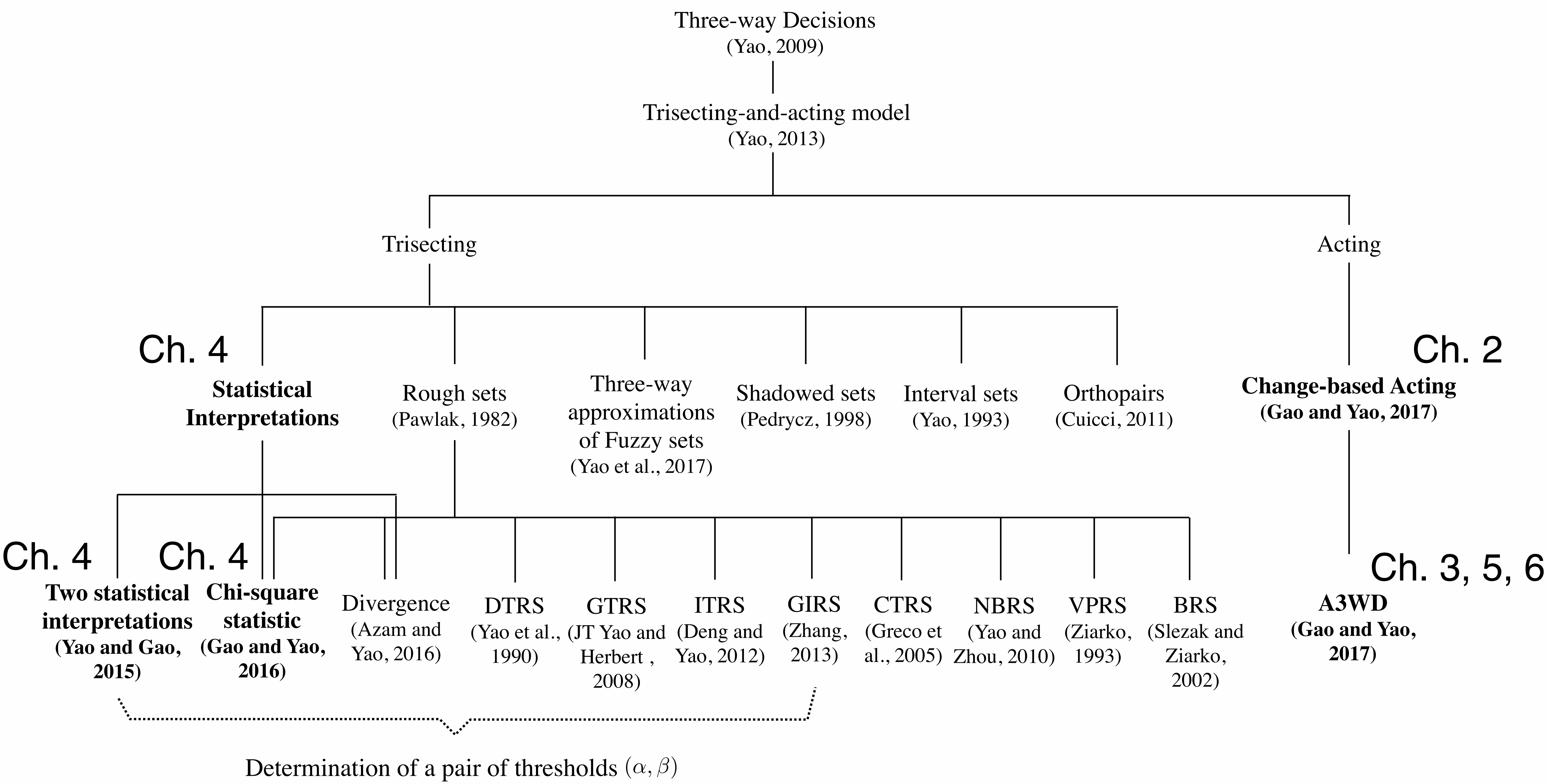
Acting

- Acting: process objects in each region, e.g.,
  - Description of concept
  - Prediction of objects
  - Transference of objects
- Transference of objects can improve the trisection quality
  - But it was not investigated in 3WD.

Motivations and Objectives

- Trisecting
  - To statistically interpret trisecting.
  - To find the optimal pair of thresholds.
- Acting
  - To model an actionable three-way decision framework with different models.
  - To further improve performance of these models.

Categorization of Three-way Decision Models



Contributions

- Presented
  - Two statistical interpretations
  - A χ^2 based method for determining the pair of thresholds
- Proposed
  - An A3WD framework with four models
  - Four actionable rule mining algorithms for these models
  - An R4 reduction framework for A3WD
  - An Addition strategy algorithm schema for reduction
  - A specific algorithm of this schema for attribute reduction and attribute-value pair reduction

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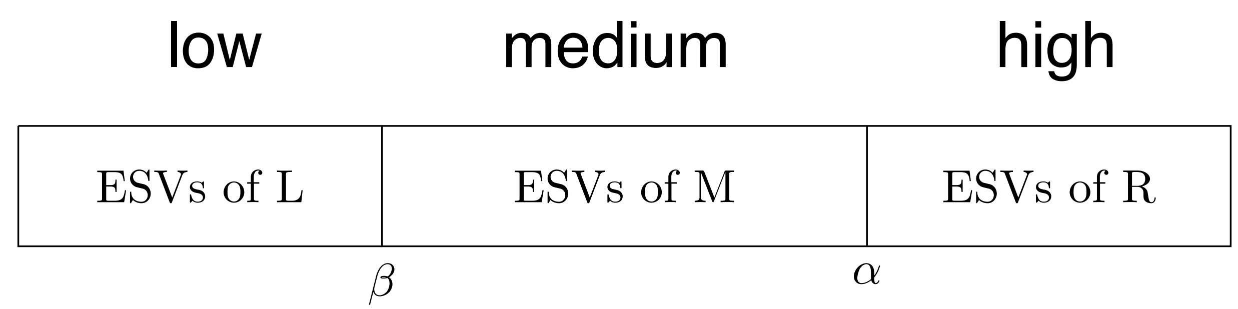
Statistical Interpretations of Trisecting

- General consideration



- Distributional characteristics in statistics
  - Median and percentile
  - Mean and standard deviation
- Two special cases of V
  - A set of non-numeric values (consider ranking)
  - A set of numeric values (arithmetic operations)

Statistical Interpretations of Trisecting (cont.)

- Interpretations through median and percentile
  - V is a set of non-numeric values, the ordering $\preccurlyeq$ only allows us to arrange objects in OB into a ranked list according to their ESVs.

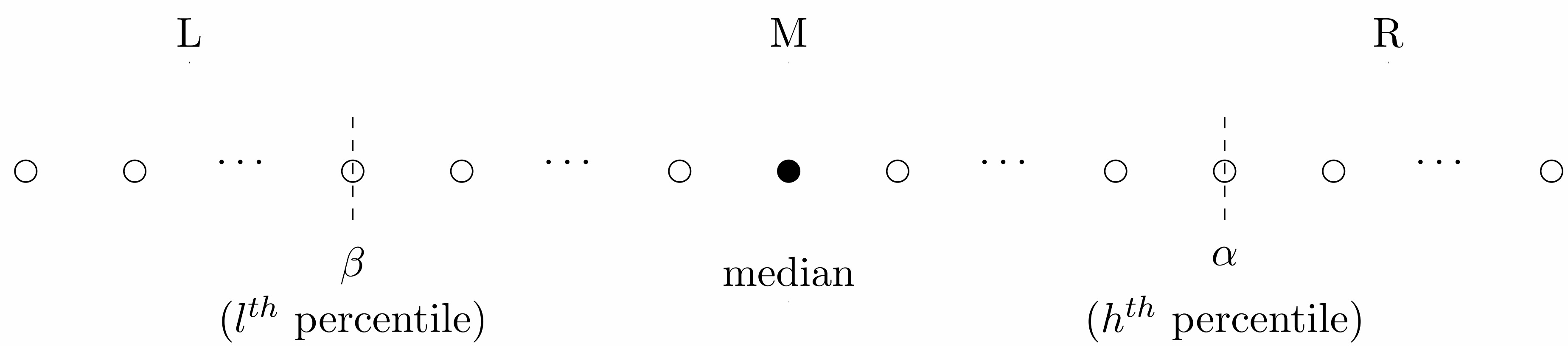


Figure 4.2: Illustration of division on rank ordered list through median and percentile.

Statistical Interpretations of Trisecting (cont.)

- Interpretations through median and percentile
  - Three regions are constructed by:

$$
\begin{aligned}
\mathrm{L}_{(\alpha,\beta)}(e) &= \{x \in OB \mid e(x) \preceq \beta\} \\
&= \{x \in OB \mid e(x) \preceq v_{\lfloor ln/100 \rfloor}\}, \\
\mathrm{M}_{(\alpha,\beta)}(e) &= \{x \in OB \mid \beta \prec e(x) \prec \alpha\} \\
&= \{x \in OB \mid v_{\lfloor ln/100 \rfloor} \prec e(x) \prec v_{\lceil hn/100 \rceil}, \\
\mathrm{R}_{(\alpha,\beta)}(e) &= \{x \in OB \mid e(x) \succeq \alpha\} \\
&= \{x \in OB \mid e(x) \succeq v_{\lceil hn/100 \rceil}\}.
\end{aligned}
$$

with

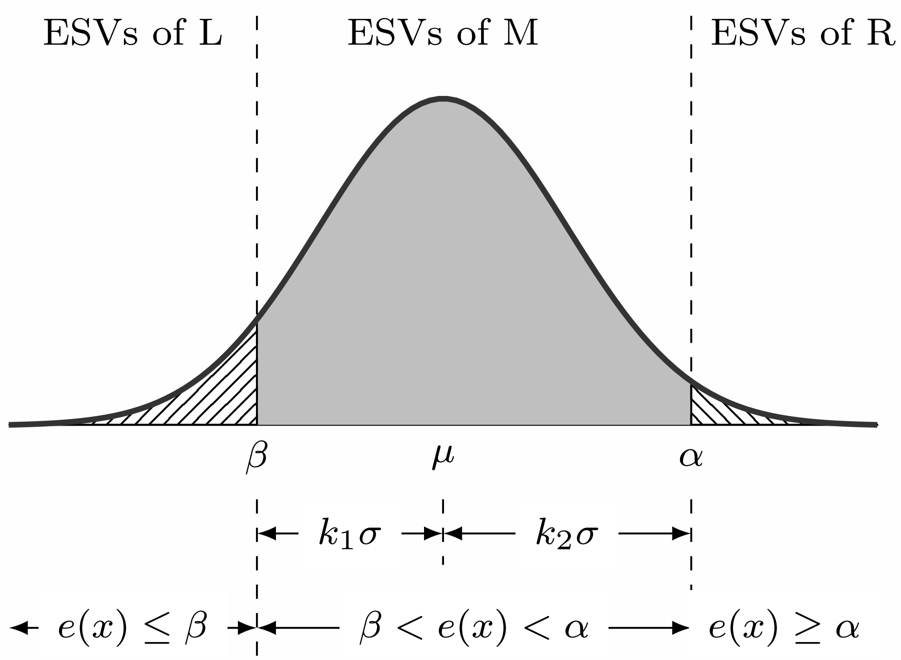
$$
\begin{aligned}
\beta &= v_{\lfloor ln/100 \rfloor}, \\
\alpha &= v_{\lceil hn/100 \rceil},
\end{aligned}
$$

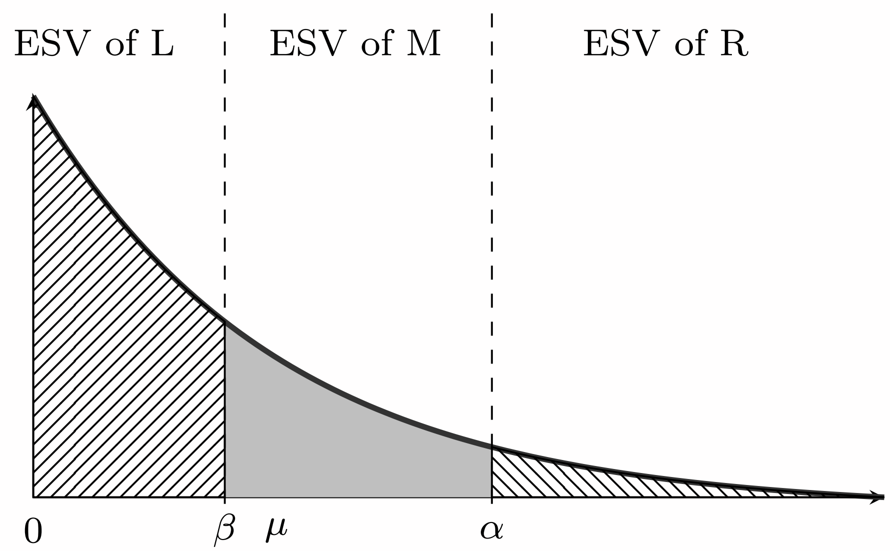
  - Example
    - Boxplot ($l = 1^{st}$ quartile, $h = 3^{rd}$ quartile) [6]

[6] P.J. Rousseeuw, I. Ruts, J. W. Tukey. The bagplot: a bivariate boxplot. The American Statistician, 53: 382-387, 1999.

Statistical Interpretations of Trisecting (cont.)

- Interpretations through mean and standard deviation
  - V is a set of numeric values, statistical measures based on arithmetic operations such as mean and standard deviation can be applied.





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Statistical Interpretations of Trisecting (cont.)

- Interpretations through mean and standard deviation
  - Three regions are constructed by:

$$
\begin{aligned}
\mathrm{L}_{(k_1,k_2)}(e) &= \{x \in OB \mid e(x) \leq \beta\} \\
&= \{x \in OB \mid e(x) \leq \mu - k_1\sigma\}, \\
\mathrm{M}_{(k_1,k_2)}(e) &= \{x \in OB \mid \beta < e(x) < \alpha\} \\
&= \{x \in OB \mid \mu - k_1\sigma < e(x) < \mu + k_2\sigma\}, \\
\mathrm{R}_{(k_1,k_2)}(e) &= \{x \in OB \mid e(x) \geq \alpha\} \\
&= \{x \in OB \mid e(x) \geq \mu + k_2\sigma\},
\end{aligned}
$$

with

$$
\begin{aligned}
\beta &= \mu - k_1\sigma, \quad k_1 \geq 0, \\
\alpha &= \mu + k_2\sigma, \quad k_2 \geq 0.
\end{aligned}
$$

  - Examples
    - Blood pressure ($k_1 = k_2 = 2$) [7]
    - Intelligence Quotient ($k_1 = k_2 = 2$) [8]

[7] C. Pater. The blood pressure “uncertainty range” - a pragmatic approach to overcome current diagnostic uncertainties (II). Current Controlled Trials in Cardiovascular Medicine, 6(1): 5, 2005.
[8] J.M. Sattler. Assessment of Children’s Intelligence. W.B. Saunders Company, Philadelphia, 1975.

Statistical Interpretations of Trisecting (cont.)

- Determining thresholds with χ^2
  - Contingency table

Table 4.1: A contingency table of three-way decision.

	$\text{POS}_{(\alpha,\beta)}(X)$	$\text{BND}_{(\alpha,\beta)}(X)$	$\text{NEG}_{(\alpha,\beta)}(X)$	Total
X	n_{XP}	n_{XB}	n_{XN}	$n_{X\cdot}$
X^C	n_{X^CP}	n_{X^CB}	n_{X^CN}	$n_{X^C\cdot}$
Total	$n_{\cdot P}$	$n_{\cdot B}$	$n_{\cdot N}$	n

  - Measurement of divergences between observation and expectation

$$\begin{aligned}
Q(\text{POS}_{(\alpha,\beta)}(X)) &= \frac{(n_{XP} - n_{X\cdot}n_{\cdot P}/n)^2}{n_{X\cdot}n_{\cdot P}/n} + \frac{(n_{X^CP} - n_{X^C\cdot}n_{\cdot P}/n)^2}{n_{X^C\cdot}n_{\cdot P}/n},\\
Q(\text{BND}_{(\alpha,\beta)}(X)) &= \frac{(n_{XB} - n_{X\cdot}n_{\cdot B}/n)^2}{n_{X\cdot}n_{\cdot B}/n} + \frac{(n_{X^CB} - n_{X^C\cdot}n_{\cdot B}/n)^2}{n_{X^C\cdot}n_{\cdot B}/n},\\
Q(\text{NEG}_{(\alpha,\beta)}(X)) &= \frac{(n_{XN} - n_{X\cdot}n_{\cdot N}/n)^2}{n_{X\cdot}n_{\cdot N}/n} + \frac{(n_{X^CN} - n_{X^C\cdot}n_{\cdot N}/n)^2}{n_{X^C\cdot}n_{\cdot N}/n}.
\end{aligned}$$

  - χ^2 as objective function and maximize it for optimal trisection

$$\begin{aligned}
Q(\pi_{(\alpha,\beta)}(X)) &= Q(\text{POS}_{(\alpha,\beta)}(X)) + Q(\text{BND}_{(\alpha,\beta)}(X)) + Q(\text{NEG}_{(\alpha,\beta)}(X))\\
&= \chi^2_{(\alpha,\beta)}.
\end{aligned}$$

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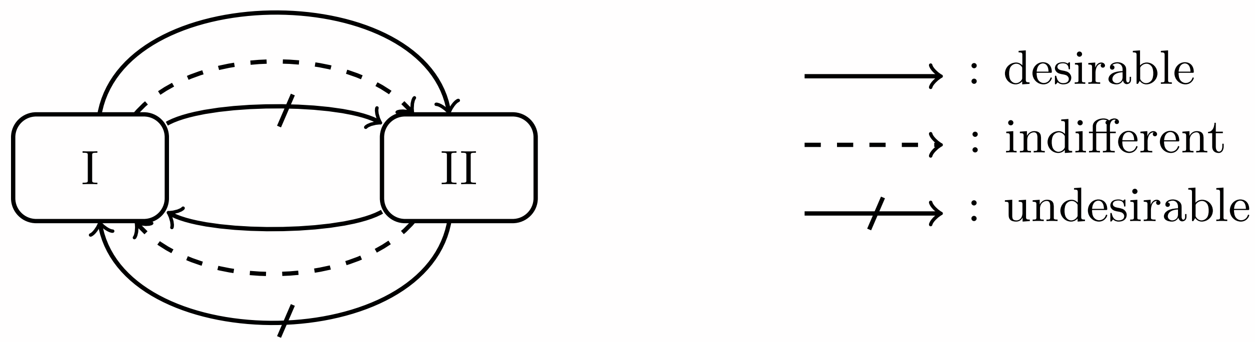
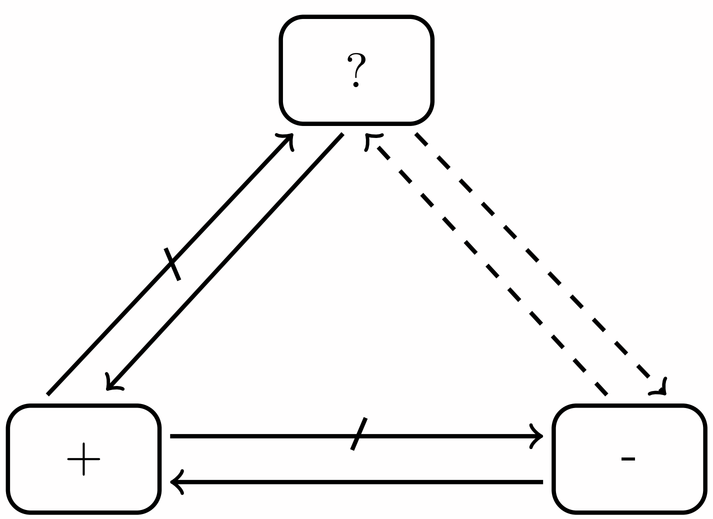
Change-based Acting

- Movements between regions



- Movement patterns



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Actionable Rule

- Categorization of attributes to A_s and A_f
- Classification rule

$$r_{[x]}: \quad \left[\bigwedge_{s\in A_s} s = I_s(x)\right] \wedge \left[\bigwedge_{f\in A_f} f = I_f(x)\right] \Rightarrow d = I_d(x),$$

$$r_{[y]}: \quad \left[\bigwedge_{s\in A_s} s = I_s(y)\right] \wedge \left[\bigwedge_{f\in A_f} f = I_f(y)\right] \Rightarrow d = I_d(y).$$

- Actionable rule (referred to as action) [9]

$$r_{[x]} \rightsquigarrow r_{[y]} : \overset{\rightsquigarrow}{\bigwedge_{f\in A_f}} I_f(x) \rightsquigarrow I_f(y), \quad subject\ to \bigwedge_{s\in A_s} I_s(x) = I_s(y)$$

[9] Z.W. Ras, A. Wieczorkowska. Action rules: how to increase profit of a company. In Proceedings of the European Conference on Principles of Data Mining and Knowledge Discovery, pp. 587-592, 2000.

Actionable Rule (cont.)

- Action(s) induce a new trisection

$$\pi \xrightarrow{\text{action(s)}} \pi'$$

- Each action brings benefit and incurs cost
  - Benefit: difference between $Q(\pi)$ and $Q(\pi')$
  - Cost: all resources required by action

Quantification the Benefits and Costs of Actions

- Three assumptions
  - (A1) Value changes among different attributes are independent.
  - (A2) All actions are independent.
  - (A3) After taking action $r_{[x]} \rightsquigarrow r_{[y]}$, $[x]$ will have the same structure of $[y]$, i.e., $Pr(X|[x])=Pr(X|[y])$.
- Based on (A1) and (A2), the action cost can be calculated:

$$C_{r_{[x]} \rightsquigarrow r_{[y]}} = |[x]| \sum_{f \in A_f} C_f(I_f(x), I_f(y))$$

- Based on (A3), the action benefit can be proven:

$$B_{r_{[x]} \rightsquigarrow r_{[y]}} = w_W \left[-b\lambda_{WP} - (|[x]| - b)\lambda_{WN} \right] + w_V \left[a\lambda_{VP} + (|[x]| - a)\lambda_{VN} \right]$$

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Four Models in Different Situations

- Model (i) requires the maximum benefit solution without cost limitation.
- Model (ii) requires the minimum cost solution to obtain the maximum benefit.

} Find bounds

- Model (iii) requires the maximum benefit solution with a limited action cost.
- Model (iv) requires the minimum action cost solution to obtain a desired benefit.

} Constrained optimization

Illustration of Four Models

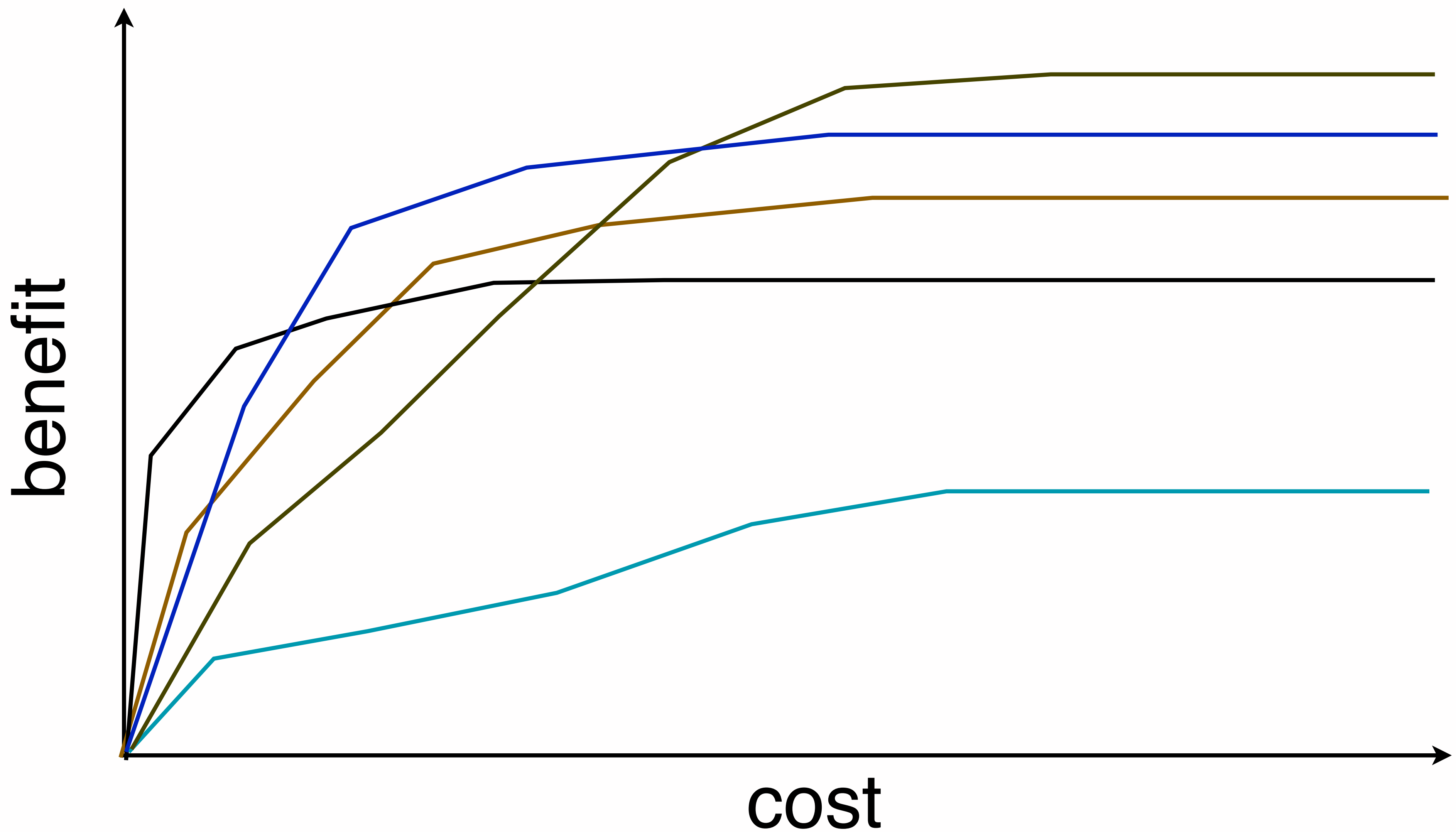

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Illustration of Four Models

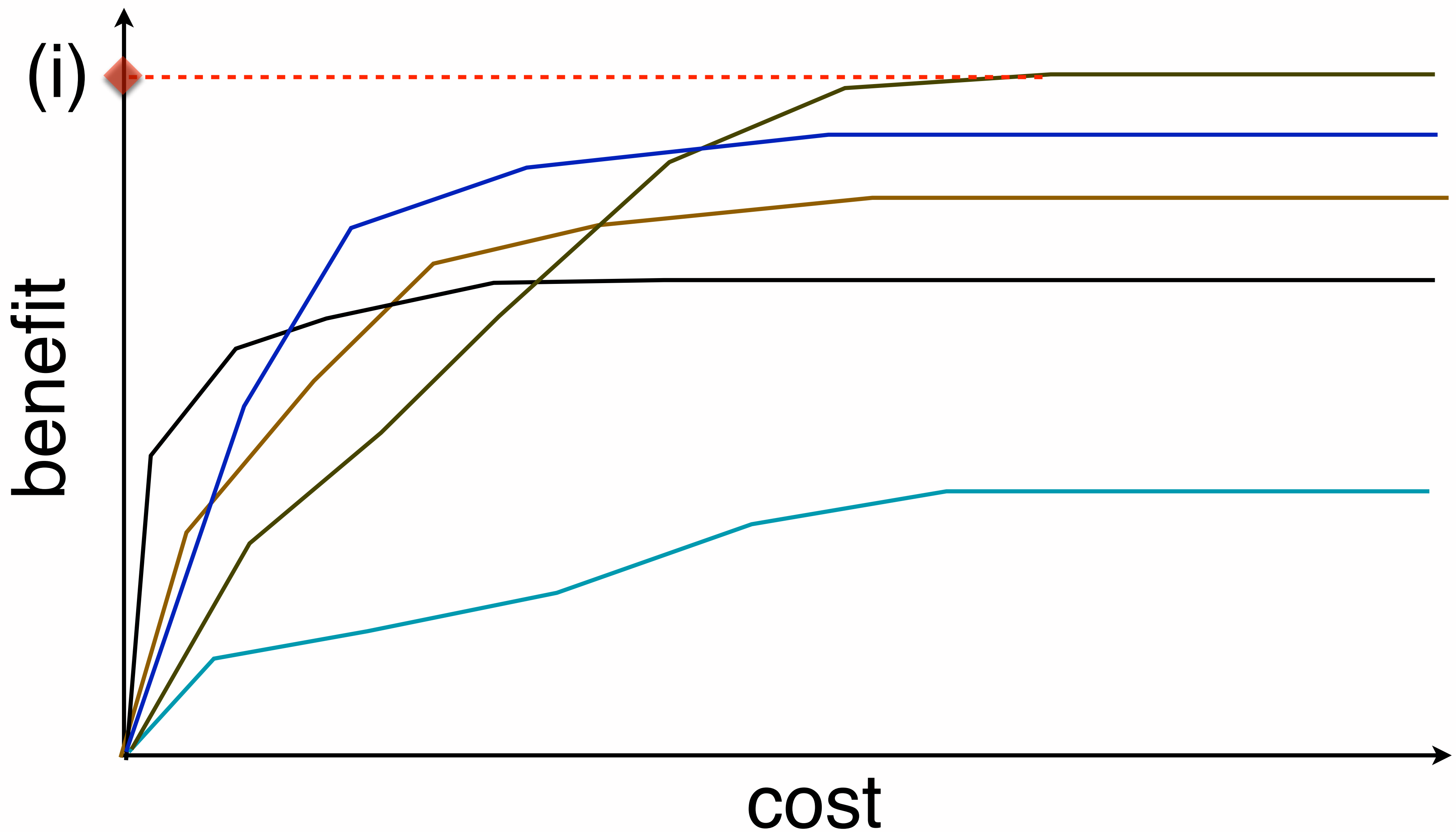

Illustration of Four Models

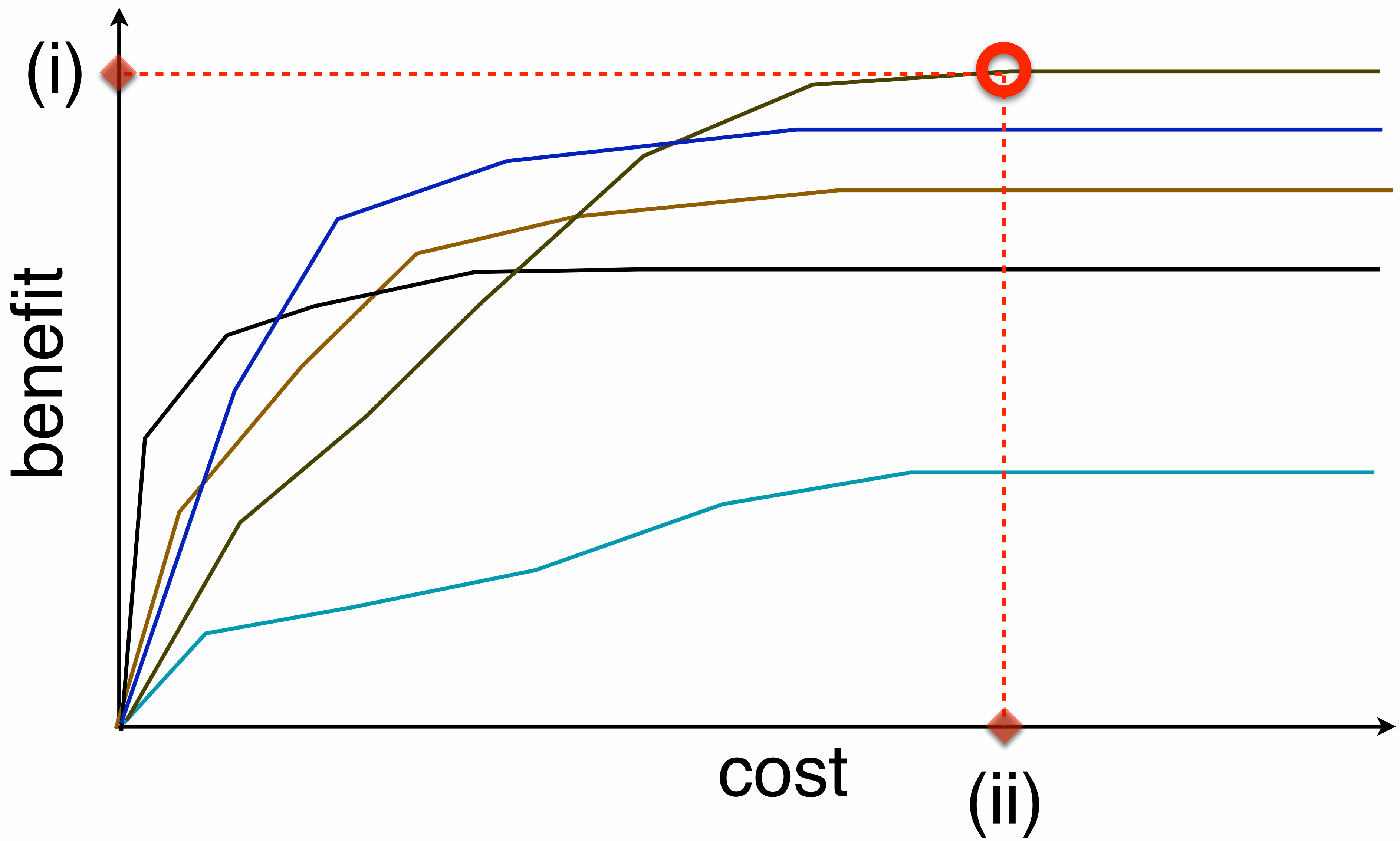

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Illustration of Four Models

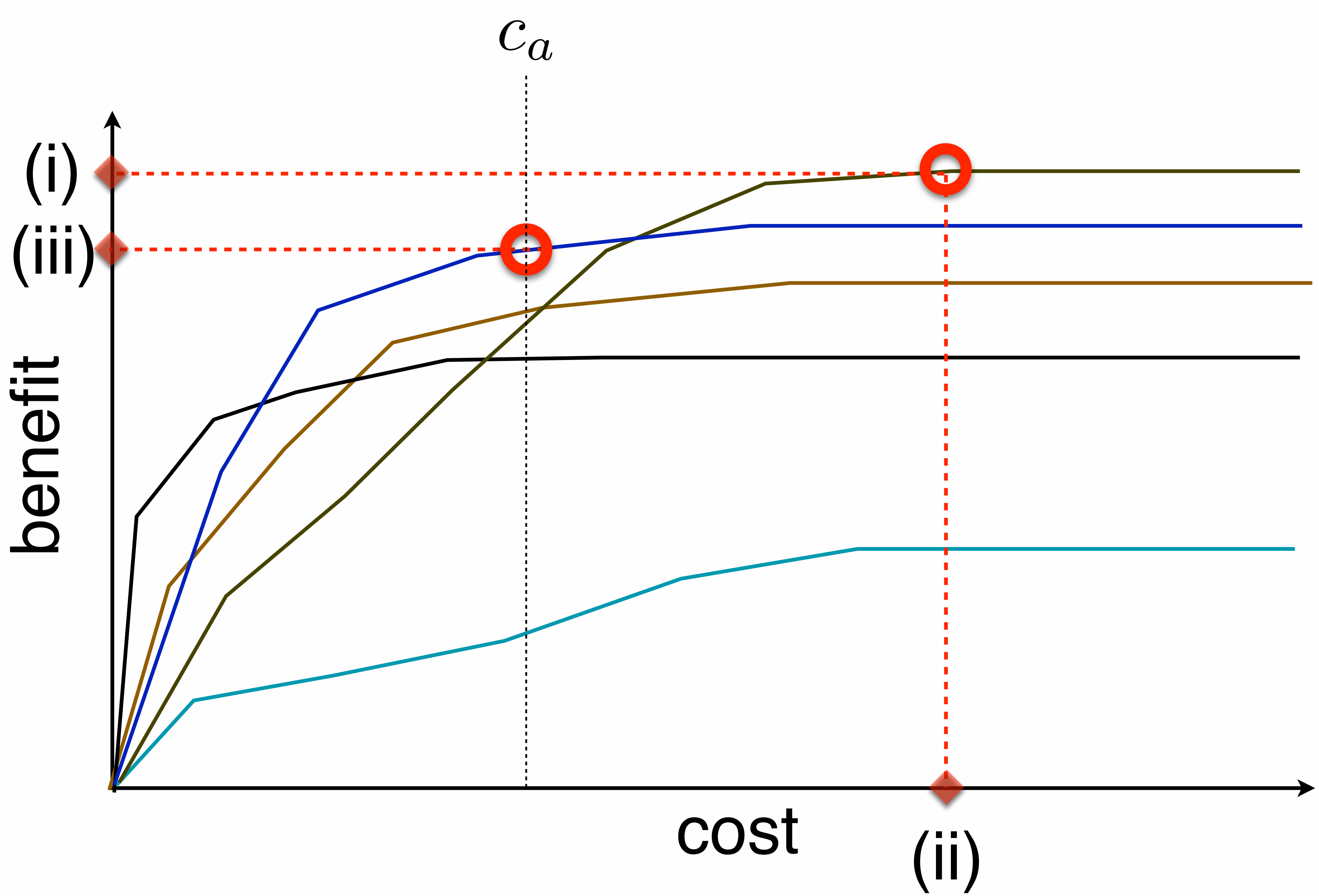

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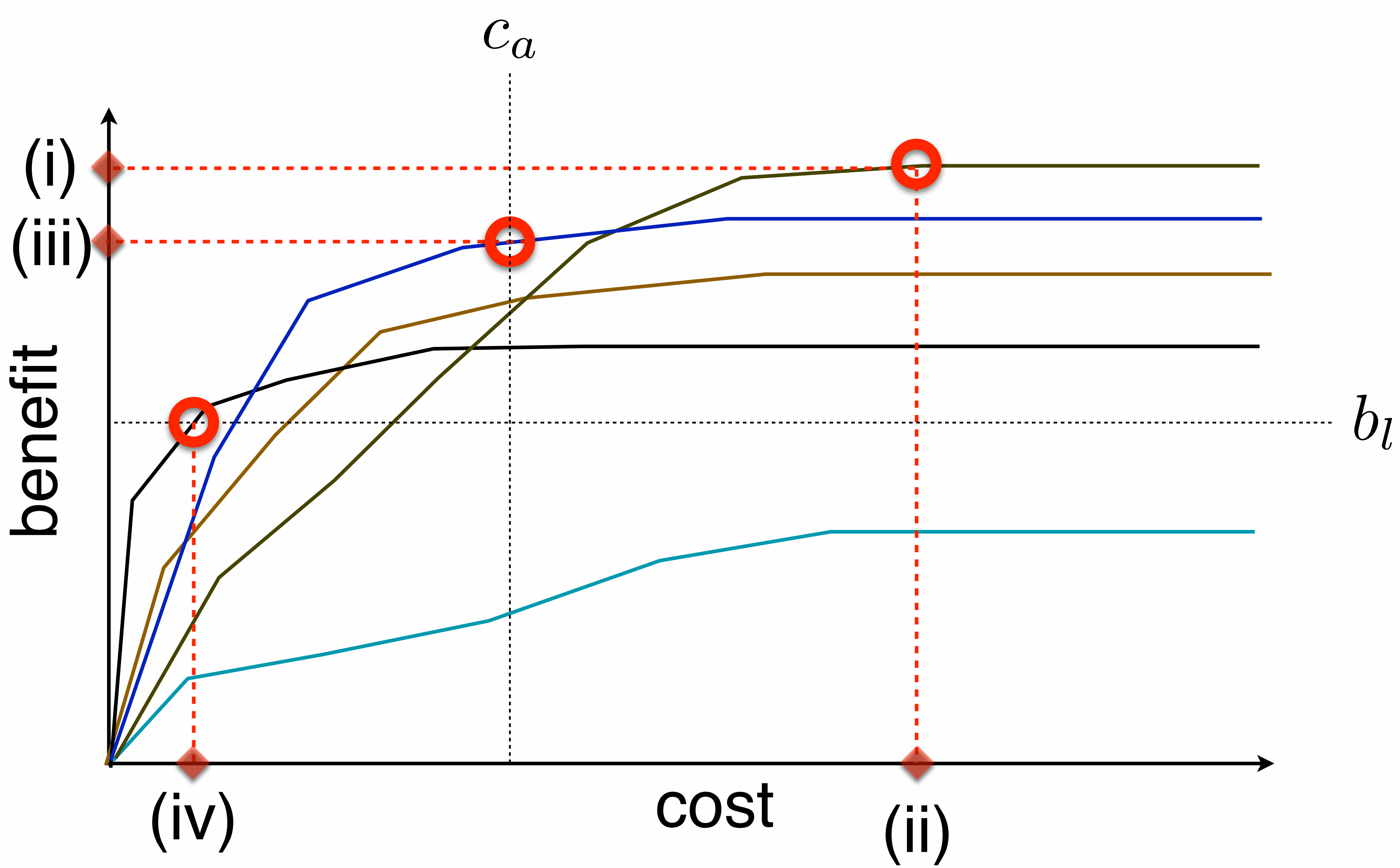
Illustration of Four Models



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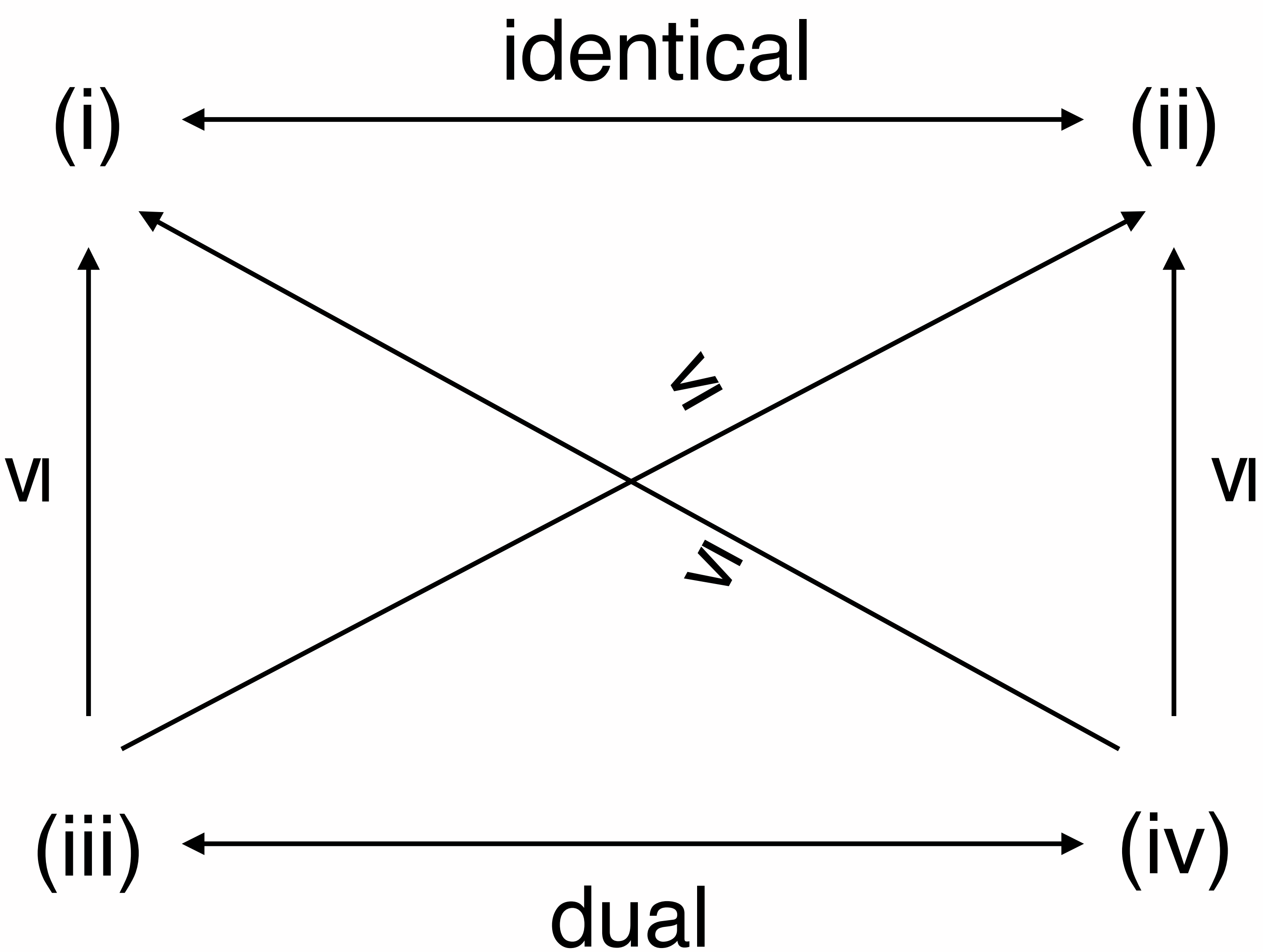
Relations of Models



Actionable Rule Mining

- Determining the bounds of benefit and cost (models (i) and (ii))
  - By previous assumptions, the maximum benefit is:

$$\bar{B} = \sum_{[x_i] \in \mathrm{SOURCE}} \max_{j=1,\ldots,n_i} \{b_{ij}\}$$

  - Time complexity: $O(|\mathrm{DES}||\mathrm{SOURCE}||A_s \cup A_f|)$.
  - It may be not unique.
  - The set of a_{ij} with minimum cost is the solution of model (ii), it may be also not unique.

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Actionable Rule Mining (cont.)

- Maximizing benefit with cost constraints (model (iii))
  - Problem analysis
    - Similar to multiple-choice knapsack problem (MCKP) [10], NP-Hard.
    - An exhaustive search has exponential time complexity.
  - Approximate solution
    - Proposed Algorithm 2, time complexity: $O(nc_am)$.

[10] D. Pisinger. Algorithms for Knapsack Problems (Ph.D. thesis). University of Copenhagen, Department of Computer Science, 1995.

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Actionable Rule Mining (cont.)

- Minimizing action cost for a desired benefit (model (iv))
  - Two algorithms are proposed
    - Algorithm 3, time complexity: $O(n\lceil \bar{C} \rceil m)$.
    - Algorithm 4, time complexity: $O(nb_l m)$.
    - Both algorithms find approximate solution.

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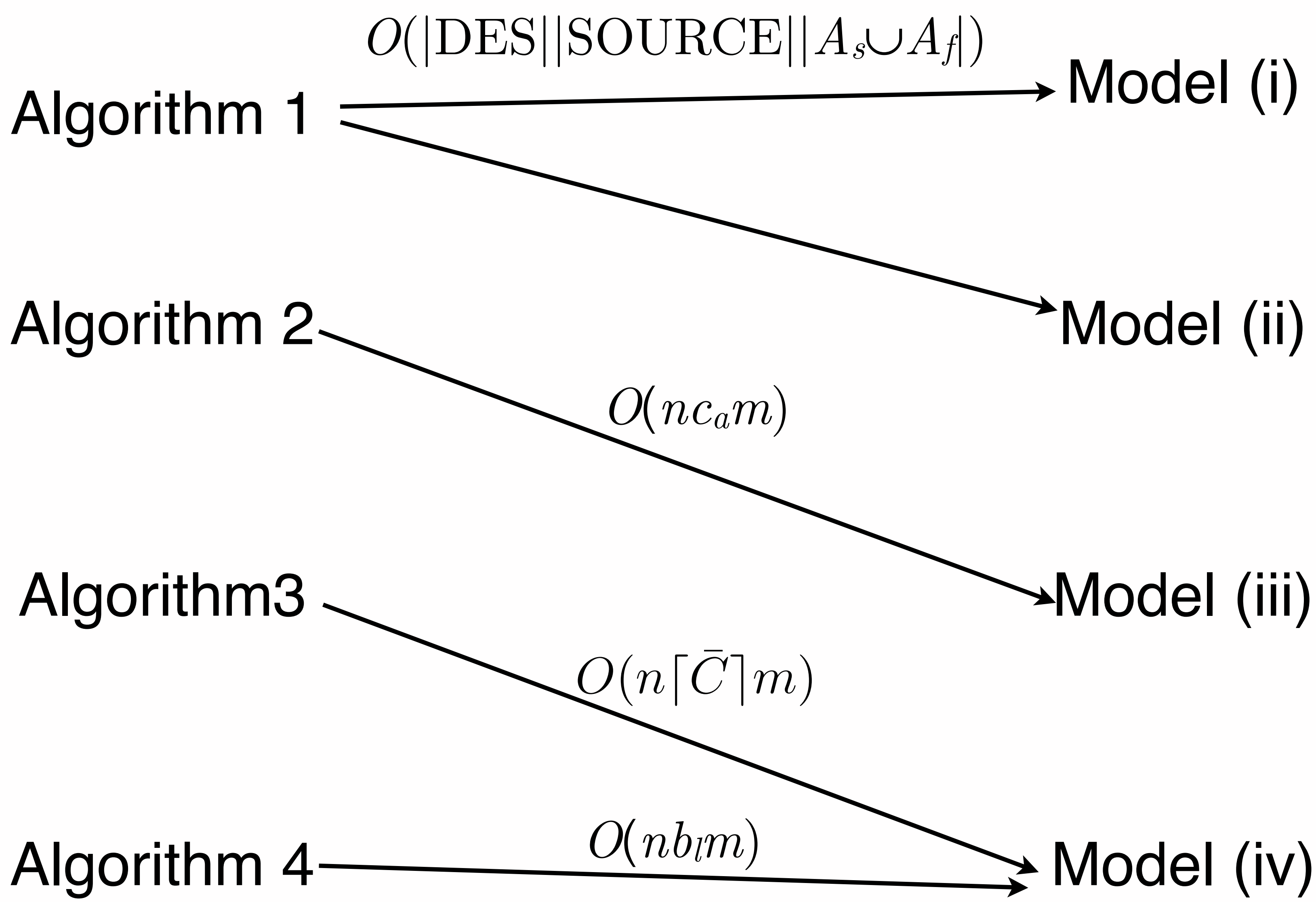
An Overview of Actionable Rule Mining Algorithms



Remove Redundancies to Improve A3WD Quality

- Motivations:
  - Increase benefit
  - Decrease cost
  - Transfer more objects
  - Decrease computation time

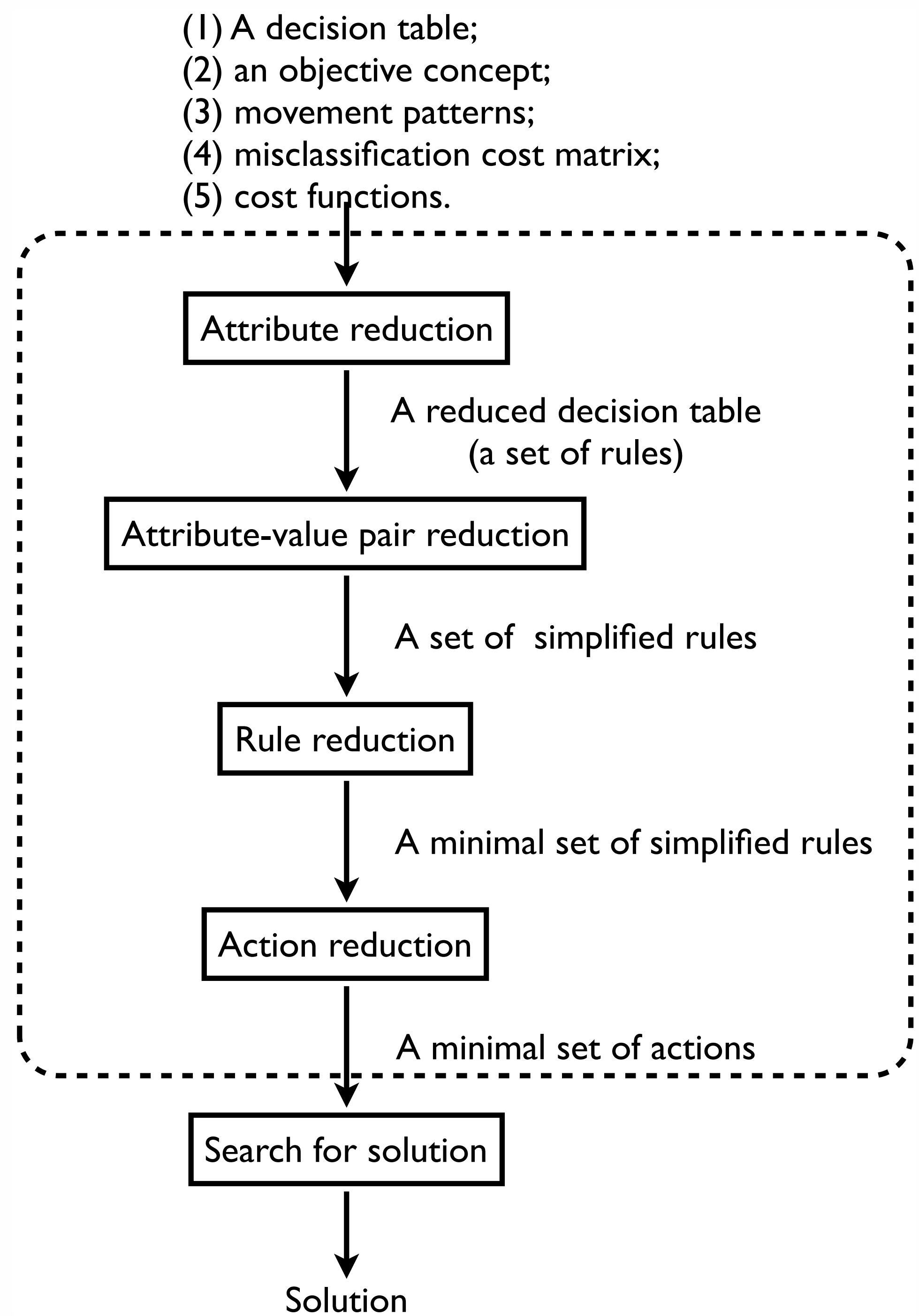


Figure 6.1: The acting procedure for actionable three-way decision making.

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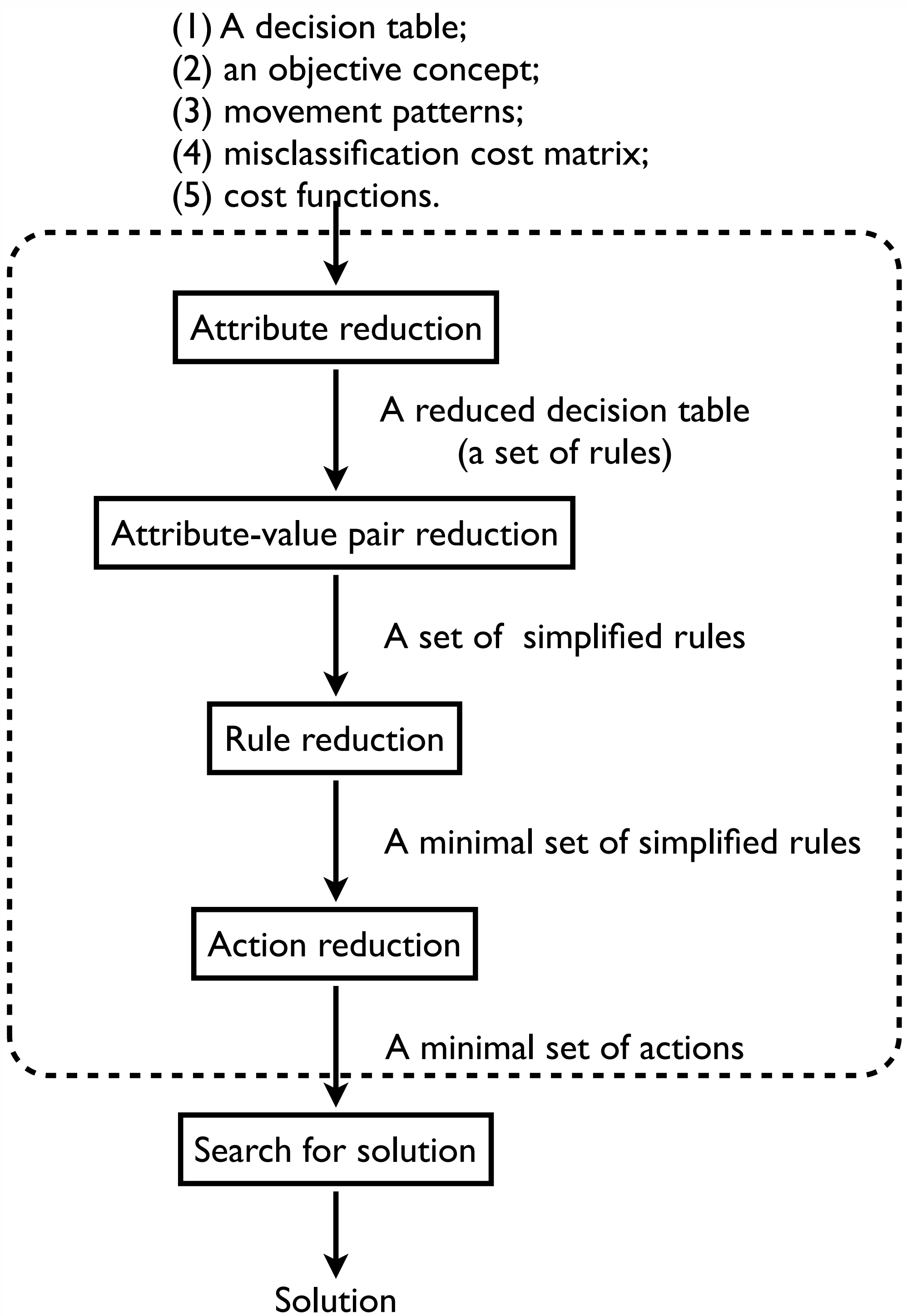
The R4 Reductions



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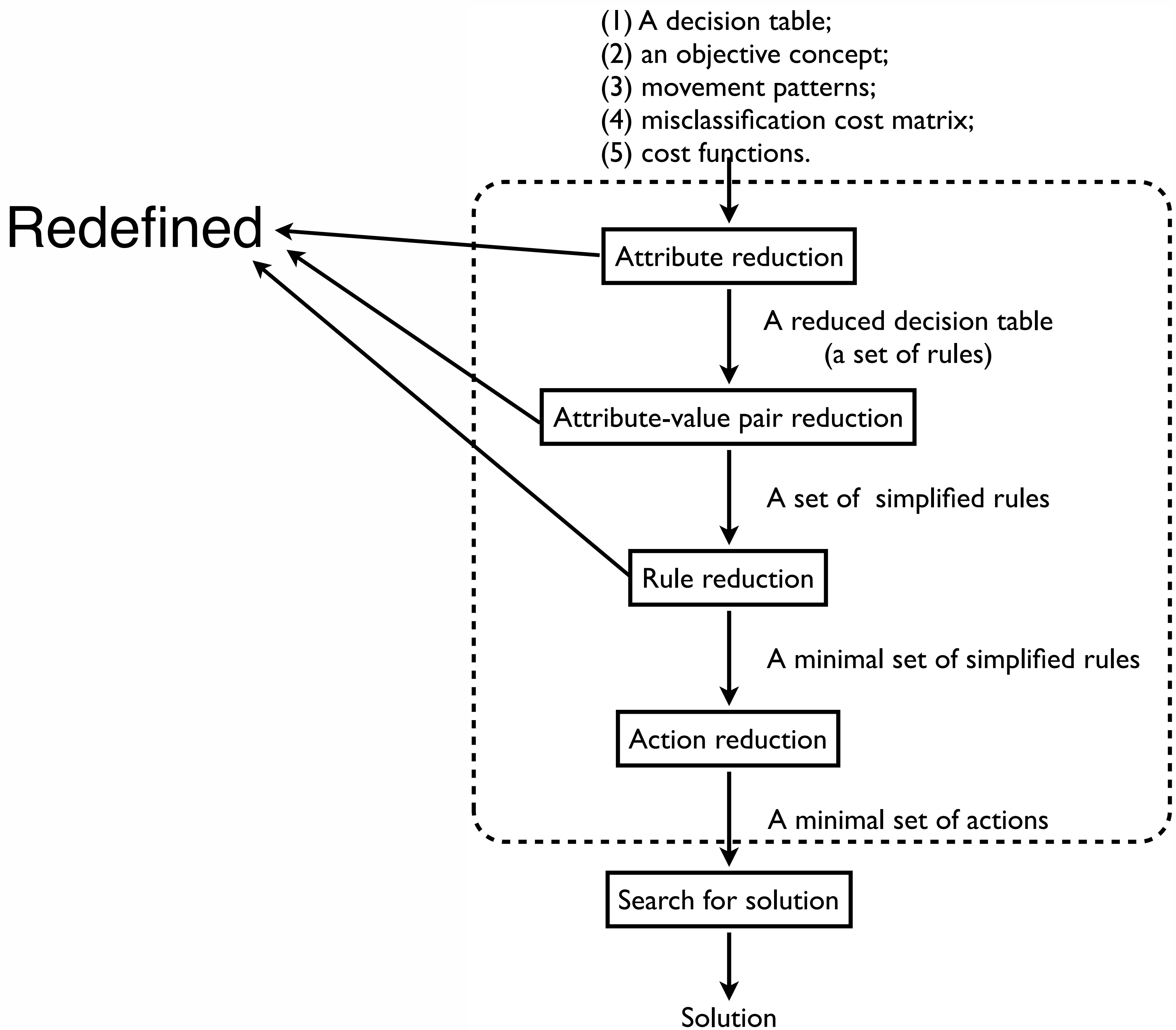
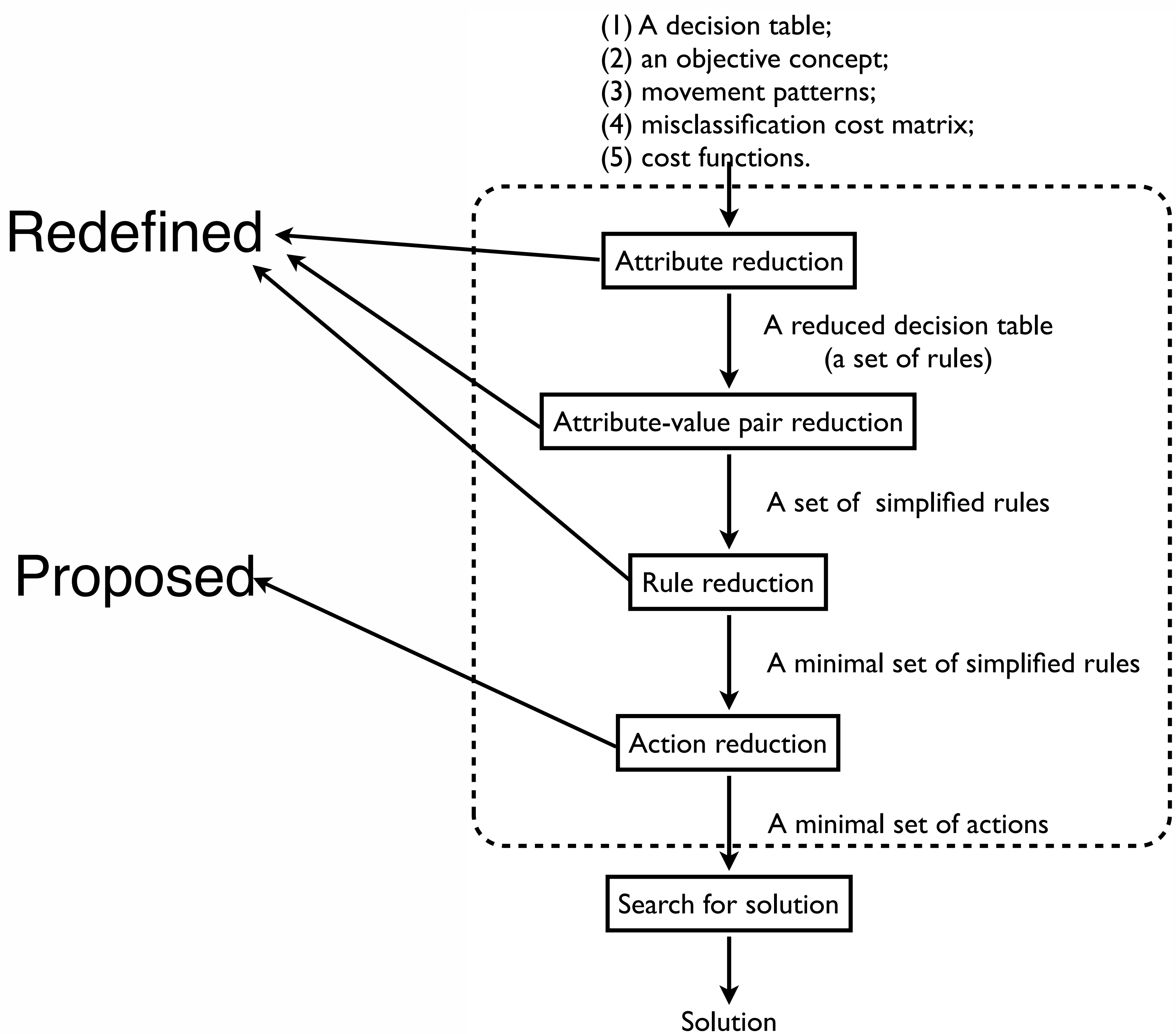


The R4 Reductions



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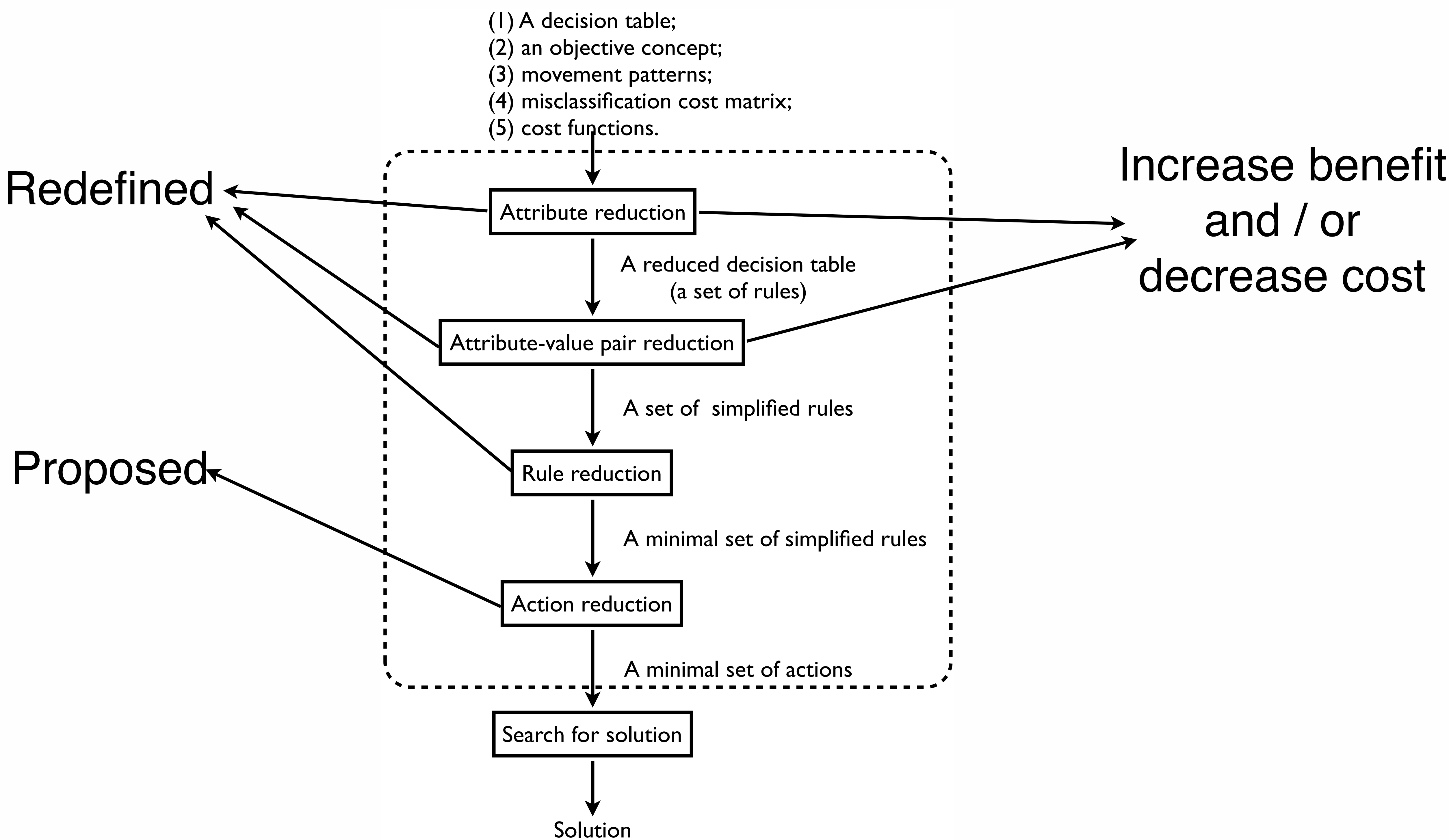
The R4 Reductions



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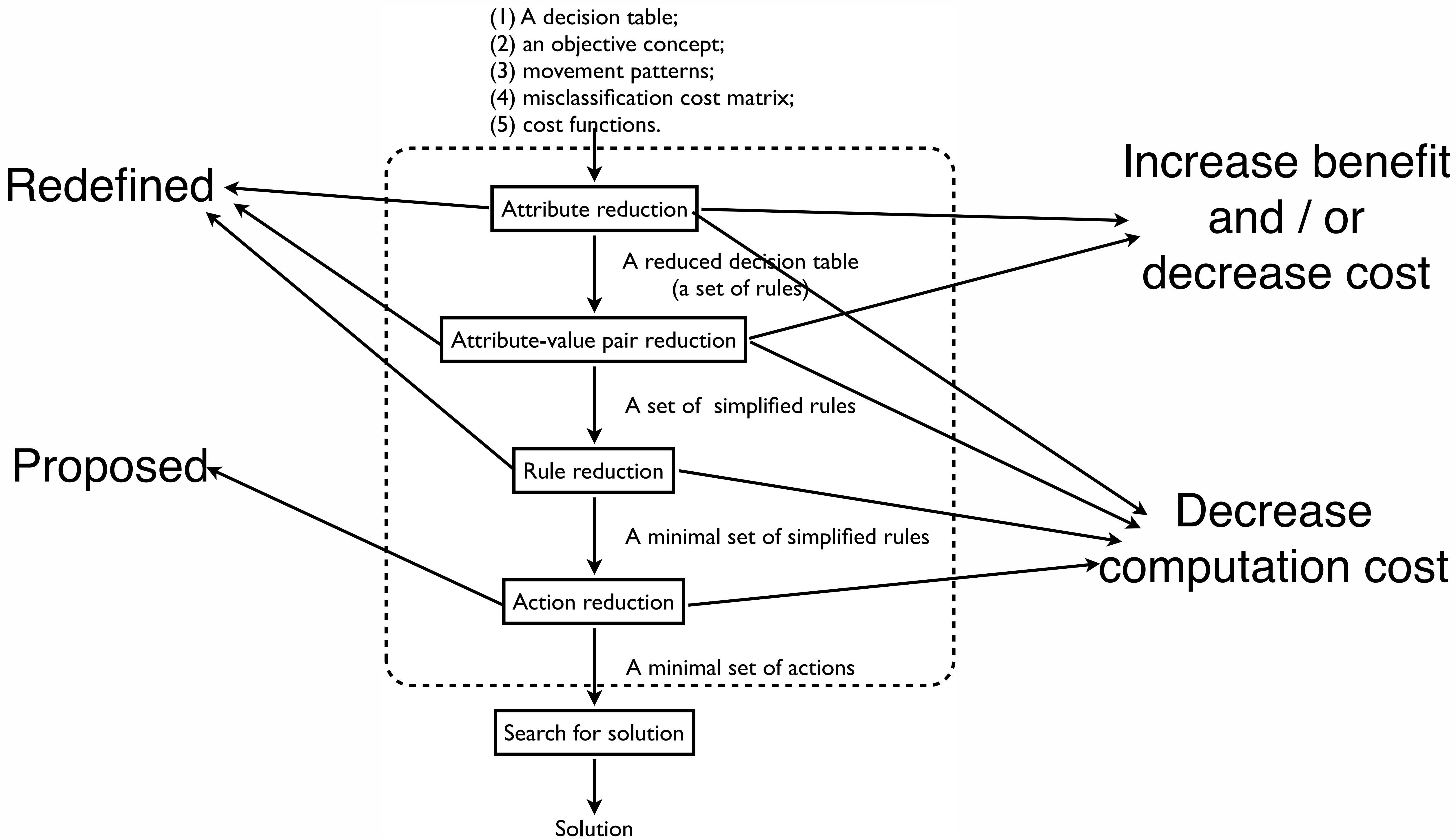
The R4 Reductions



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Attribute Reduction in A3WD

- Redefine attribute reduct to remove irrelevant attributes without changing trisection.

Definition 6.1 *An attribute set* $R \subseteq (A_s \cup A_f)$ *from a decision table* S *is called a* ***relative attribute reduct*** *of* S *with respect to the mapping* τ *if* R *satisfies the following two conditions:*

$$\text{(s1)} \quad \text{IND}(R \mid \tau) = \text{IND}(A_s \cup A_f \mid \tau);$$

$$\text{(n1)} \quad \forall a \in R, \ \text{IND}(R - \{a\} \mid \tau) \neq \text{IND}(A_s \cup A_f \mid \tau).$$

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Attribute-value Pair Reduction in A3WD

- Also called rule simplification.
- It simplifies the left-hand side of a classification rule by removing redundant attribute-value pairs without losing any classification power of the rule.

Definition 6.8 *Given a row* $d([x],[y_i]), i = 1,\ldots,n$ *of a decision matrix, let* $M = \{d([x],[y_i])\}$, *let* $AV = \bigcup_{i=1,\ldots,n} d([x],[y_i])$ *be the set of all attribute-value pairs in this row.* $R \subseteq AV$ *is an* ***attribute-value pair reduct*** *if it satisfies the following two conditions:*

$$\text{(s3)} \quad \forall d([x],[y_i]) \in M,\ R \cap d([x],[y_i]) \neq \emptyset;$$

$$\text{(n3)} \quad \forall a \in R, \exists d([x],[z]) \in M,\ (R - \{a\}) \cap d([x],[z]) = \emptyset.$$

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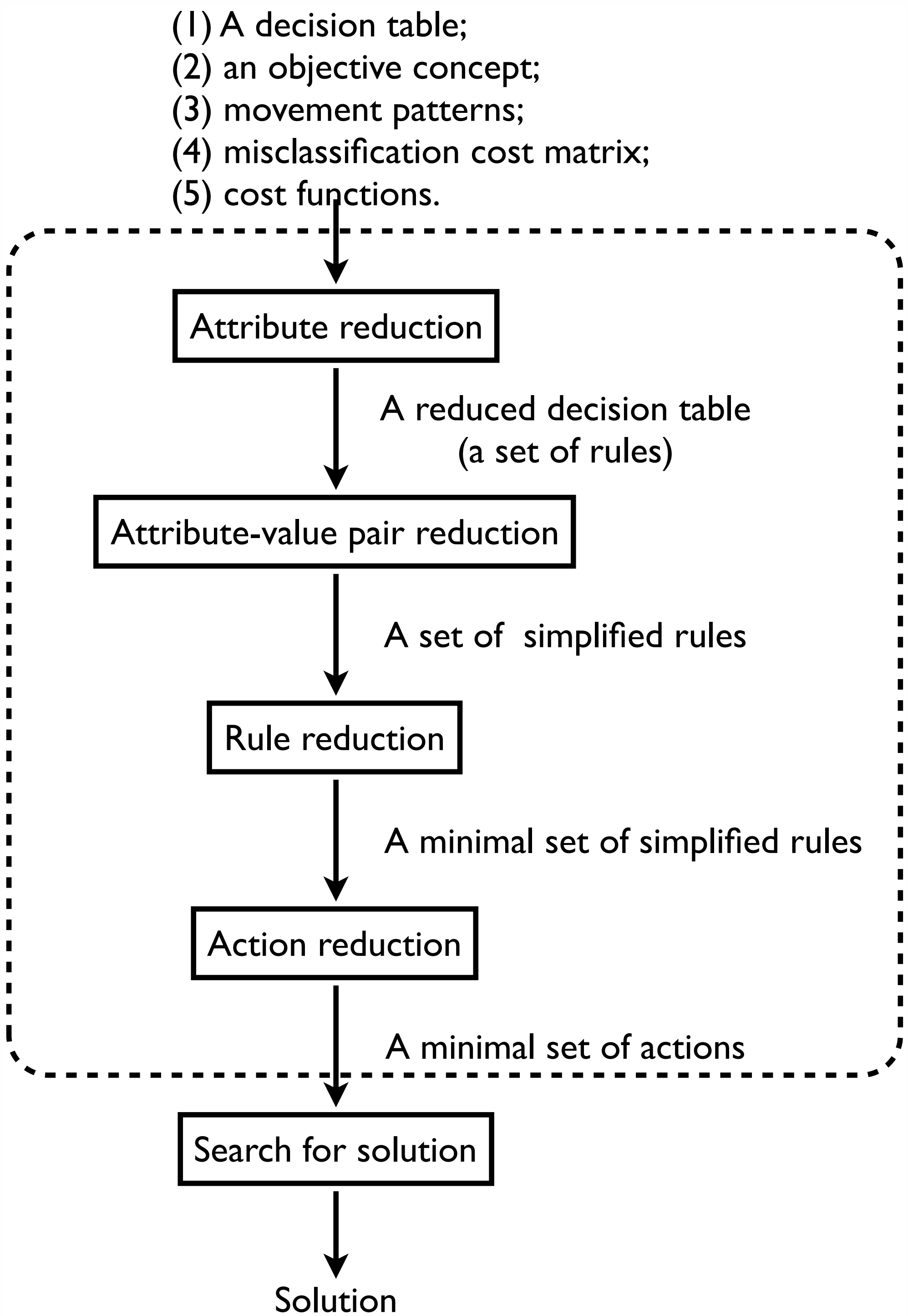
An Addition Strategy Reduction Schema



[11] Y.Y. Yao, Y. Zhao. Discernibility matrix simplification for constructing attribute reducts. Information Sciences, 179: 867-882, 2009.
[12] A. Skowron, C. Rauszer. The discernibility matrices and functions in information systems. Intelligent Decision Support, 11: 331-362, 1992.
[13] W. Ziarko, N. Shan. A method for computing all maximally general rules in attribute-value systems. Computational Intelligence, 12(2): 223-234, 1996.

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An Addition Strategy Reduction Schema

Three-way categorization of attributes

↓

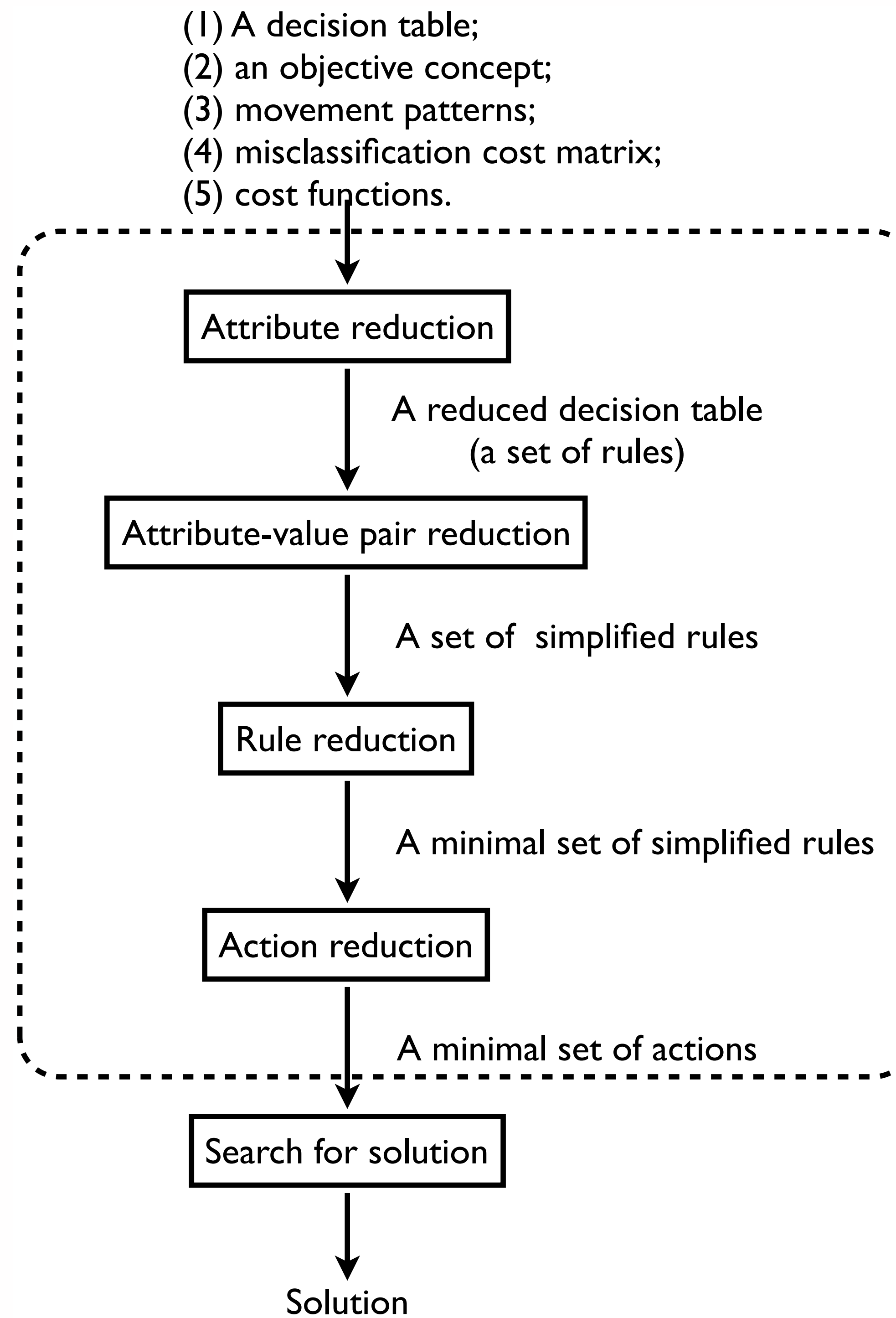
Lemma 6.1 [11]

↓

Lemma 6.2

↓

Lemma 6.3



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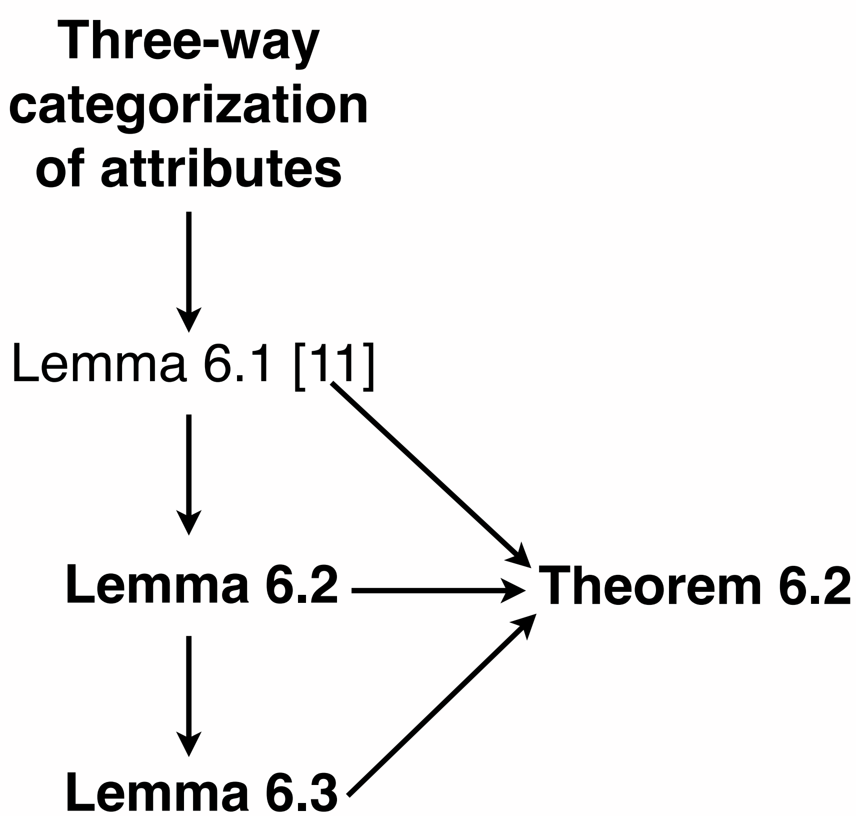
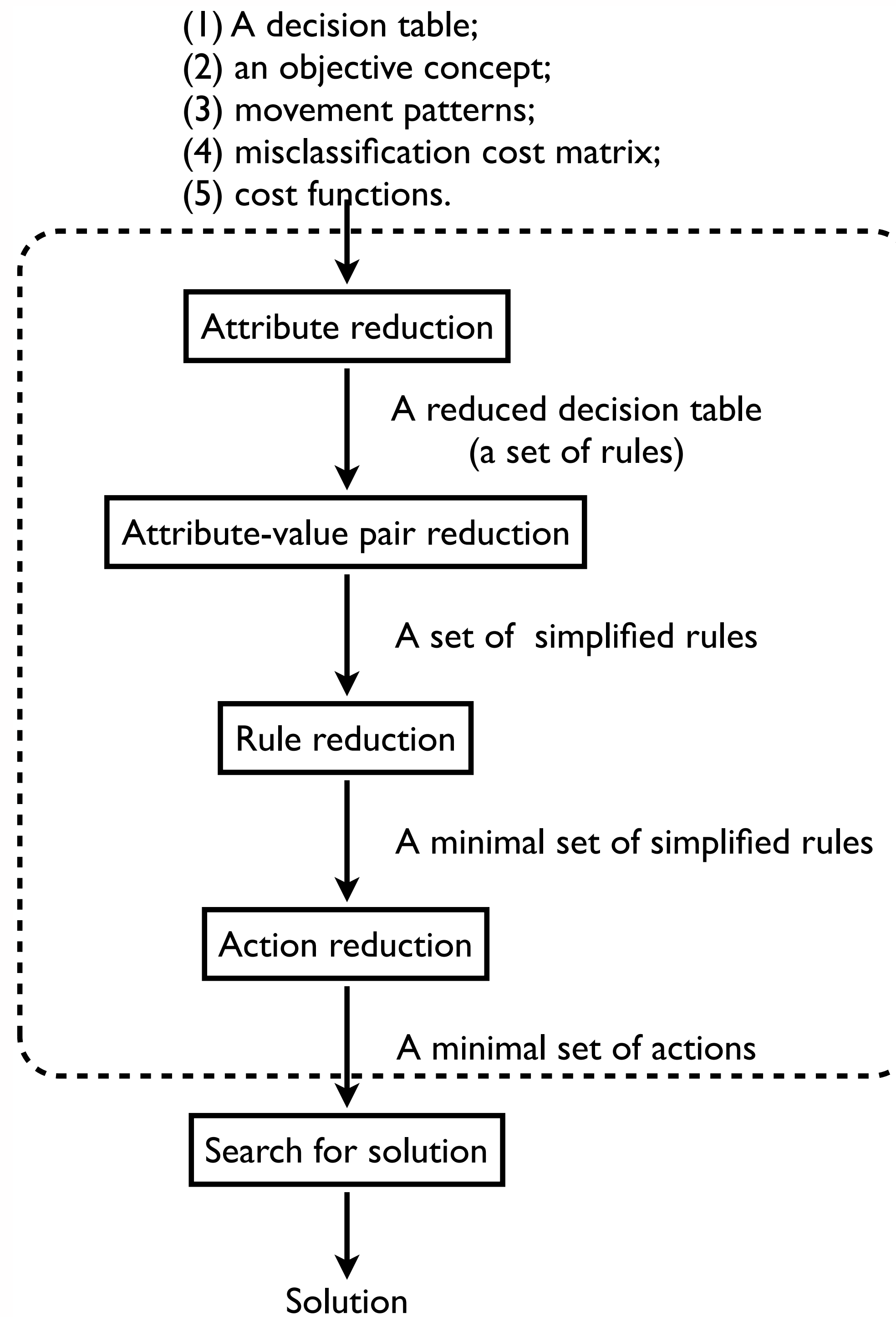
An Addition Strategy Reduction Schema





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[12] A. Skowron, C. Rauszer. The discernibility matrices and functions in information systems. Intelligent Decision Support, 11: 331-362, 1992.
[13] W. Ziarko, N. Shan. A method for computing all maximally general rules in attribute-value systems. Computational Intelligence, 12(2): 223-234, 1996.

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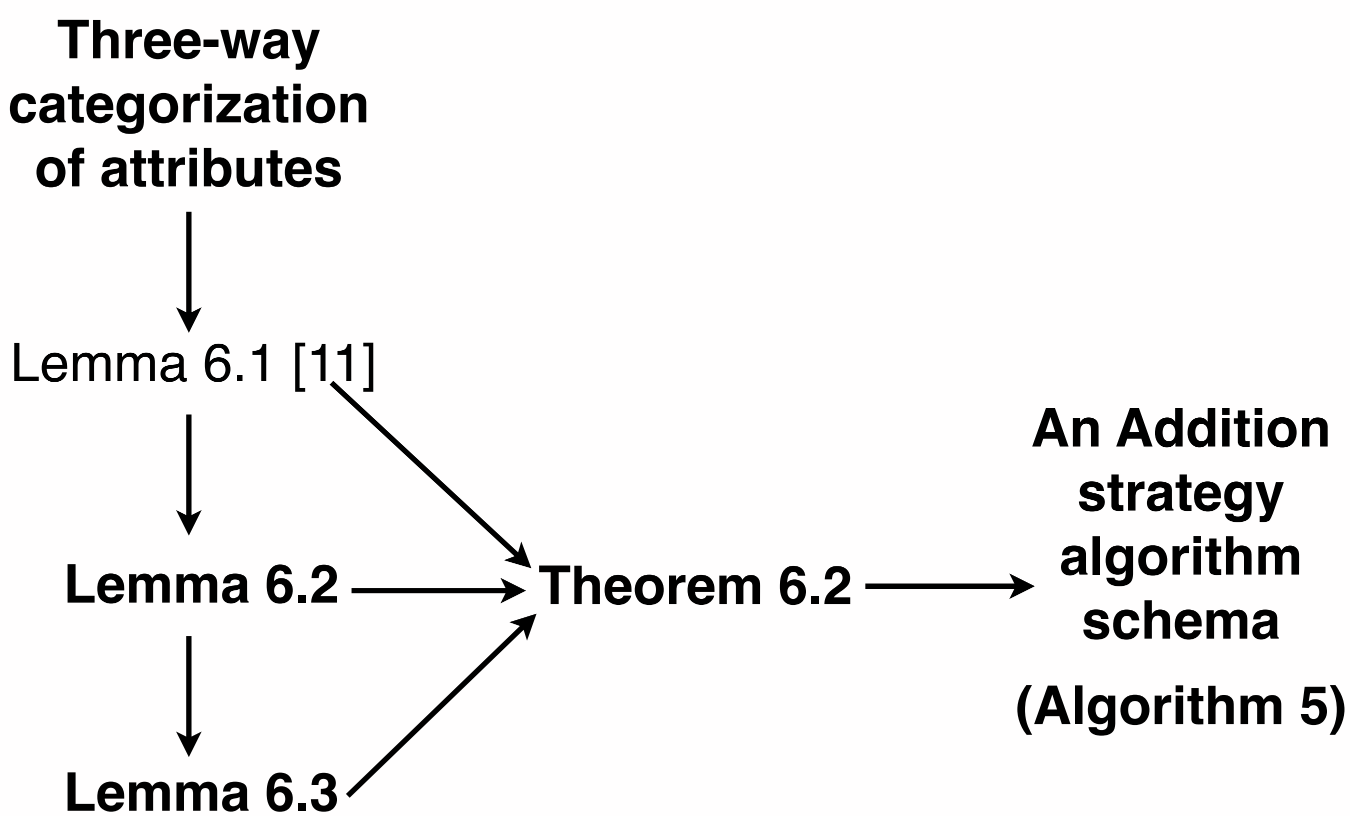

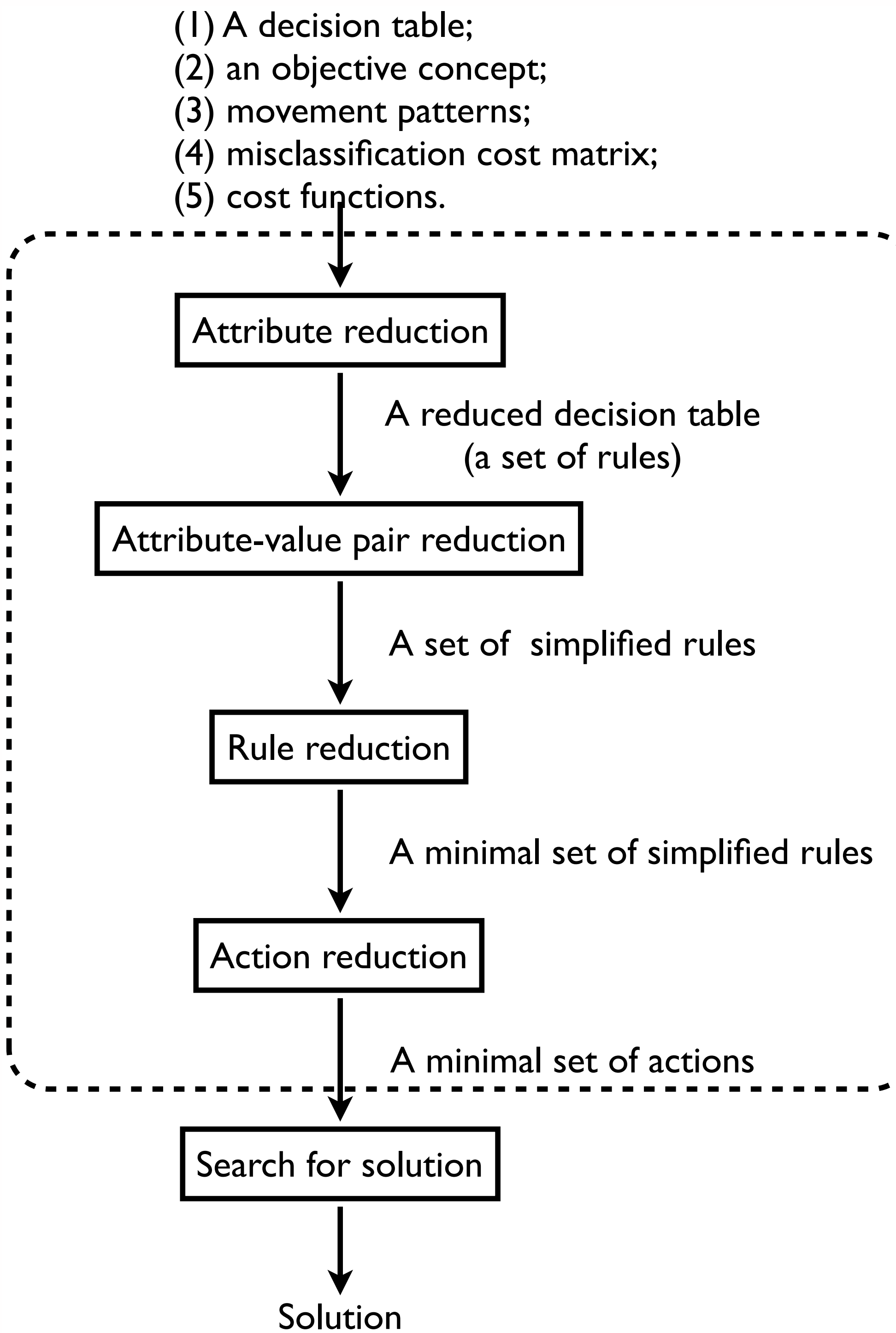

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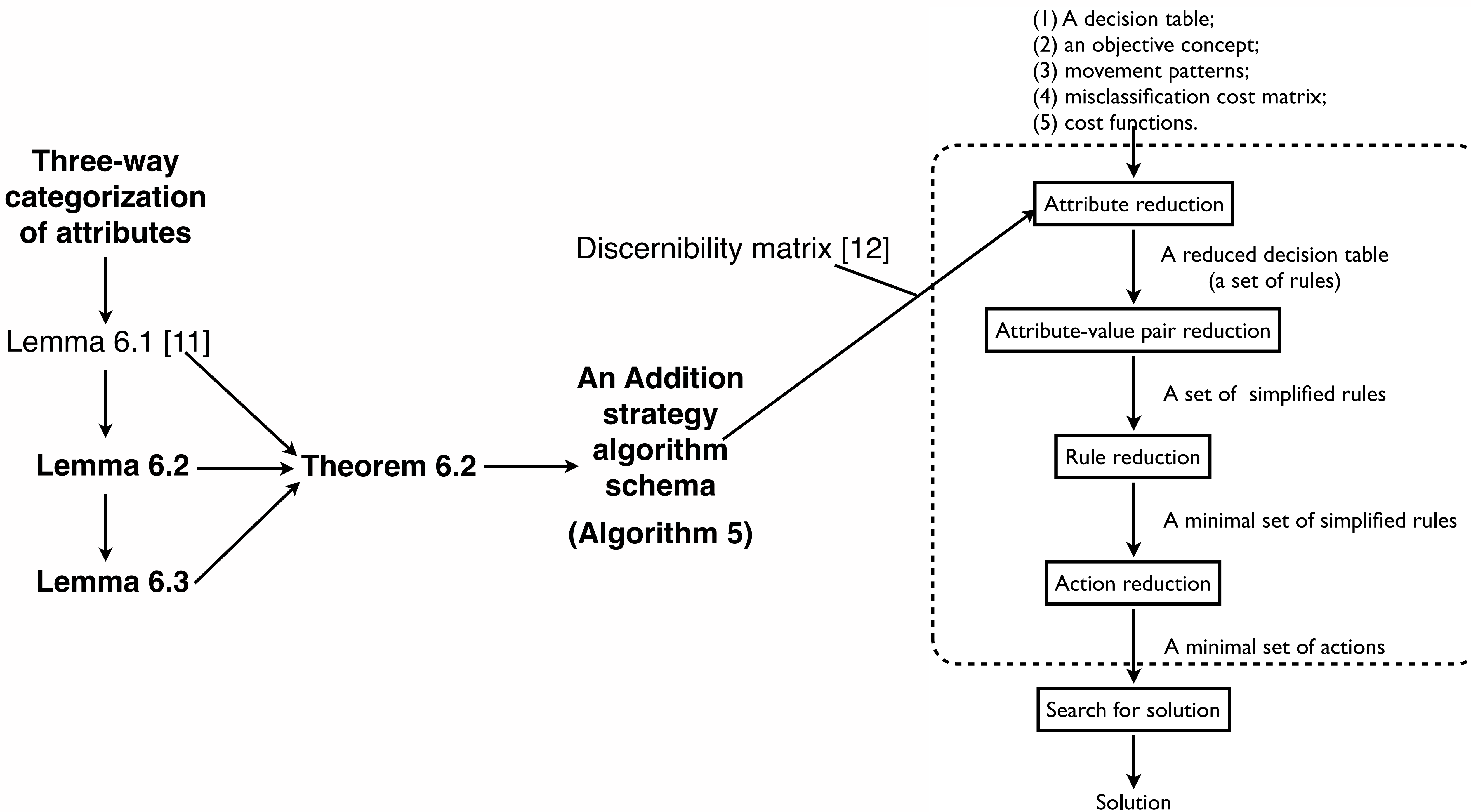

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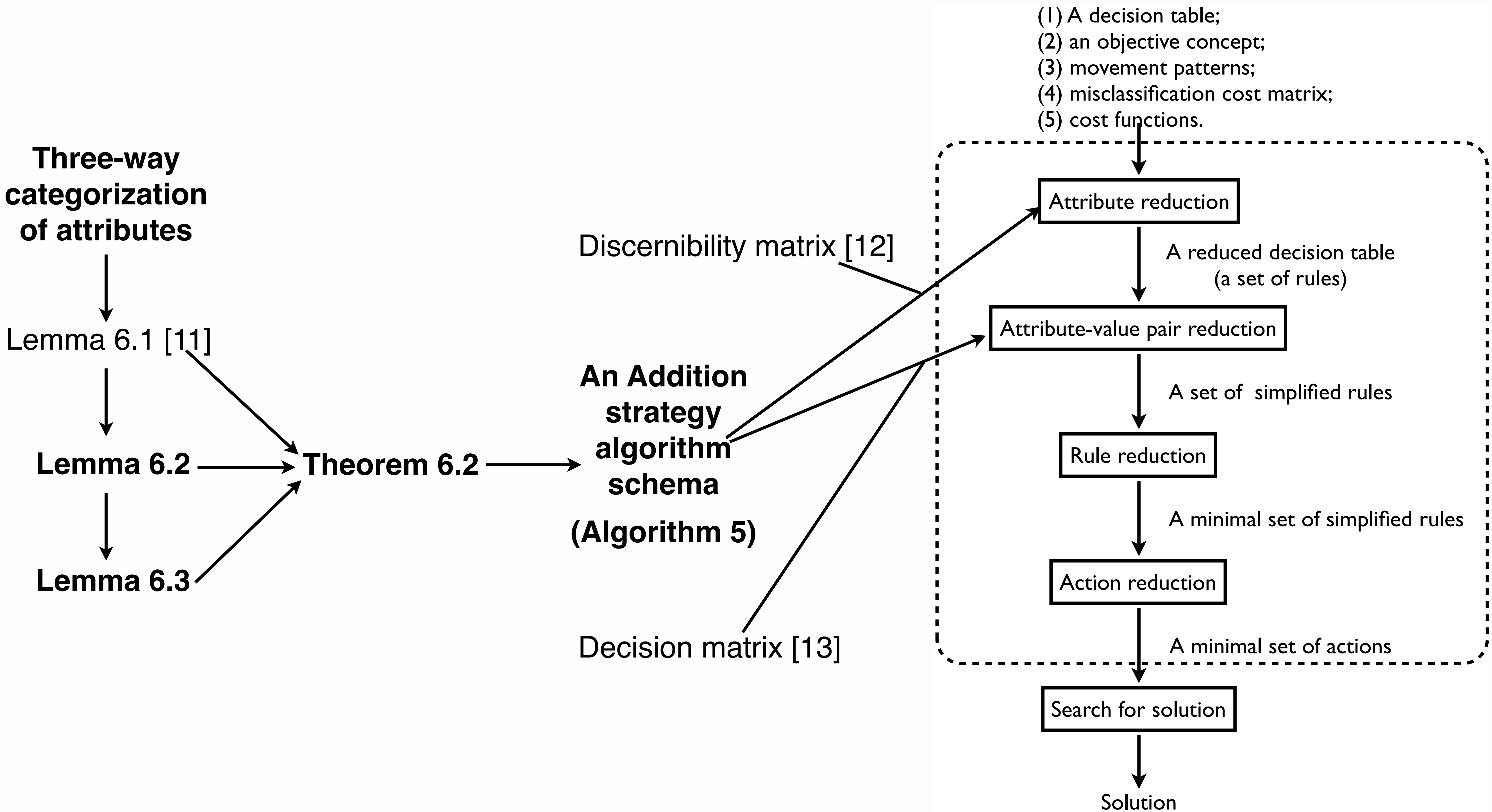

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An Addition Strategy Reduction Schema (cont.)

- Advantages:
  - Easier to understand.
  - Adopts more heuristic information, can produce better reduct.
  - More efficient than other methods when $|AT|$ is large and $|R|$ is small.
  - Many algorithms can be designed based on it.
- An algorithm instance
  - Proposed Algorithm 6, time complexity: $O(|M|^2|AT|)$.

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Rule Reduction in A3WD

- Rule reduct redefined as:

Definition 6.9 *Given an equivalence class* $[x] \subseteq S_F$, $r_{[x]}$ *is a* ***redundant rule*** *if for any desirable action* $r_{[y_i]} \rightsquigarrow r_{[x]}, [y_i] \subseteq S_U$, *there exists a desirable action* $r_{[y_i]} \rightsquigarrow r_{[z]}, [z] \neq [x], [z] \subseteq S_F$, *such that the benefit of* $r_{[y_i]} \rightsquigarrow r_{[z]}$ *is greater than or equal to the benefit of* $r_{[y_i]} \rightsquigarrow r_{[x]}$ *and the cost of* $r_{[y_i]} \rightsquigarrow r_{[z]}$ *is less than or equal to the cost of* $r_{[y_i]} \rightsquigarrow r_{[x]}$.

- The computation cost is very high, infeasible in practice.
- Special case: duplicated rule reduction
  - Time complexity: $O(n^2)$, n is the number of rules.
  - Algorithm is trivial and skipped in the thesis.

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Action Reduction in A3WD

- If an action a_1 that transfers $[x]$ has higher cost and less benefit than another action a_2, then a_1 is redundant:

Definition 6.10 *Given an action $r_{[x]} \rightsquigarrow r_{[y]}$ that transfers $[x]$, its cost and benefit are c and b, respectively. $r_{[x]} \rightsquigarrow r_{[y]}$ is a* ***redundant action*** *if*

$$\exists r_{[x]} \rightsquigarrow r_{[y_i]}, c \geq c_i \text{ and } b \leq b_i, \tag{6.14}$$

where c_i and b_i are the cost and benefit of $r_{[x]} \rightsquigarrow r_{[y_i]}$, respectively.

- The Algorithm 7 is designed for action reduction
  - Time complexity: $O(|OB|^3)$.

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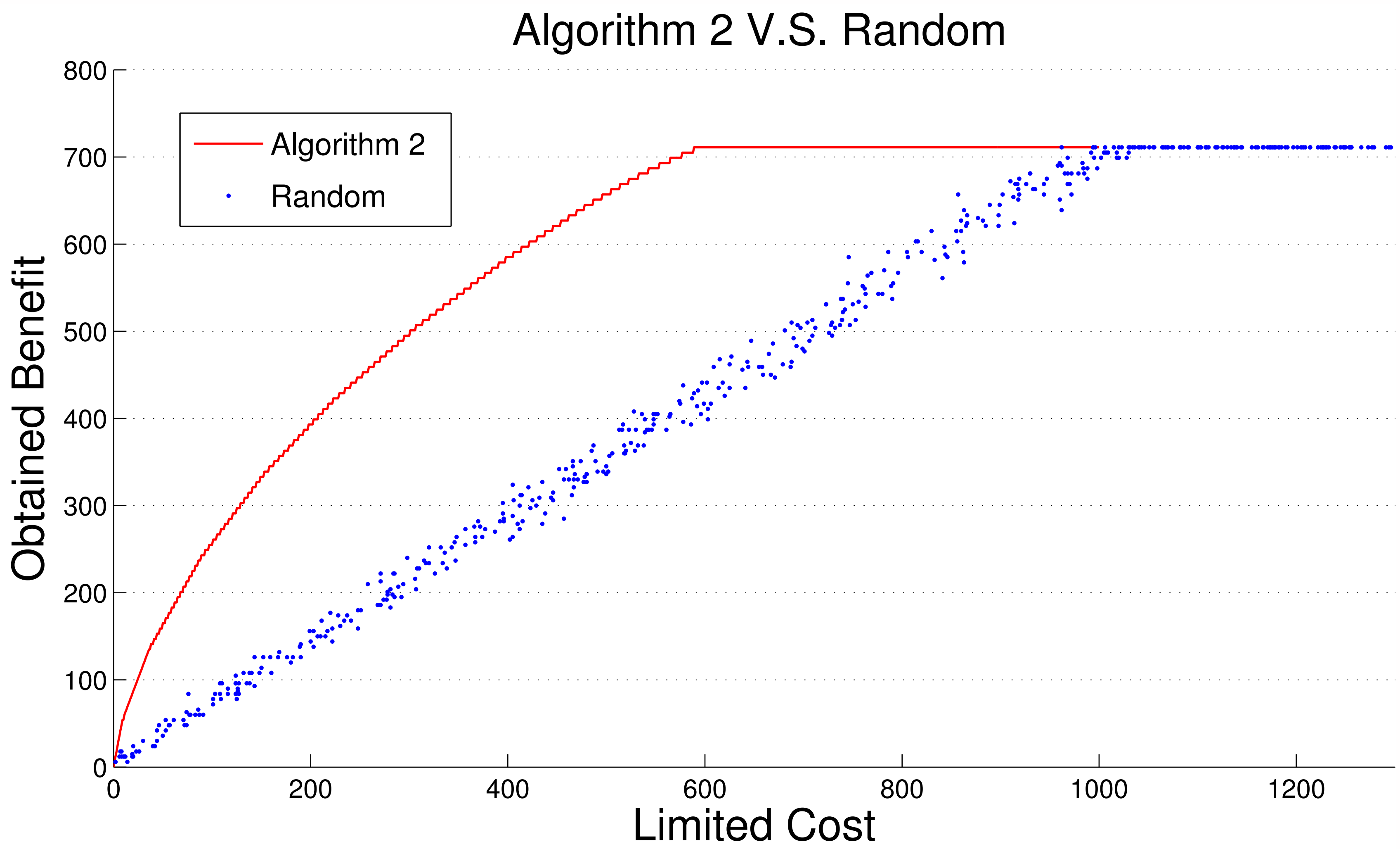
Experimental Results

- Comparison between Algorithm 2 and random



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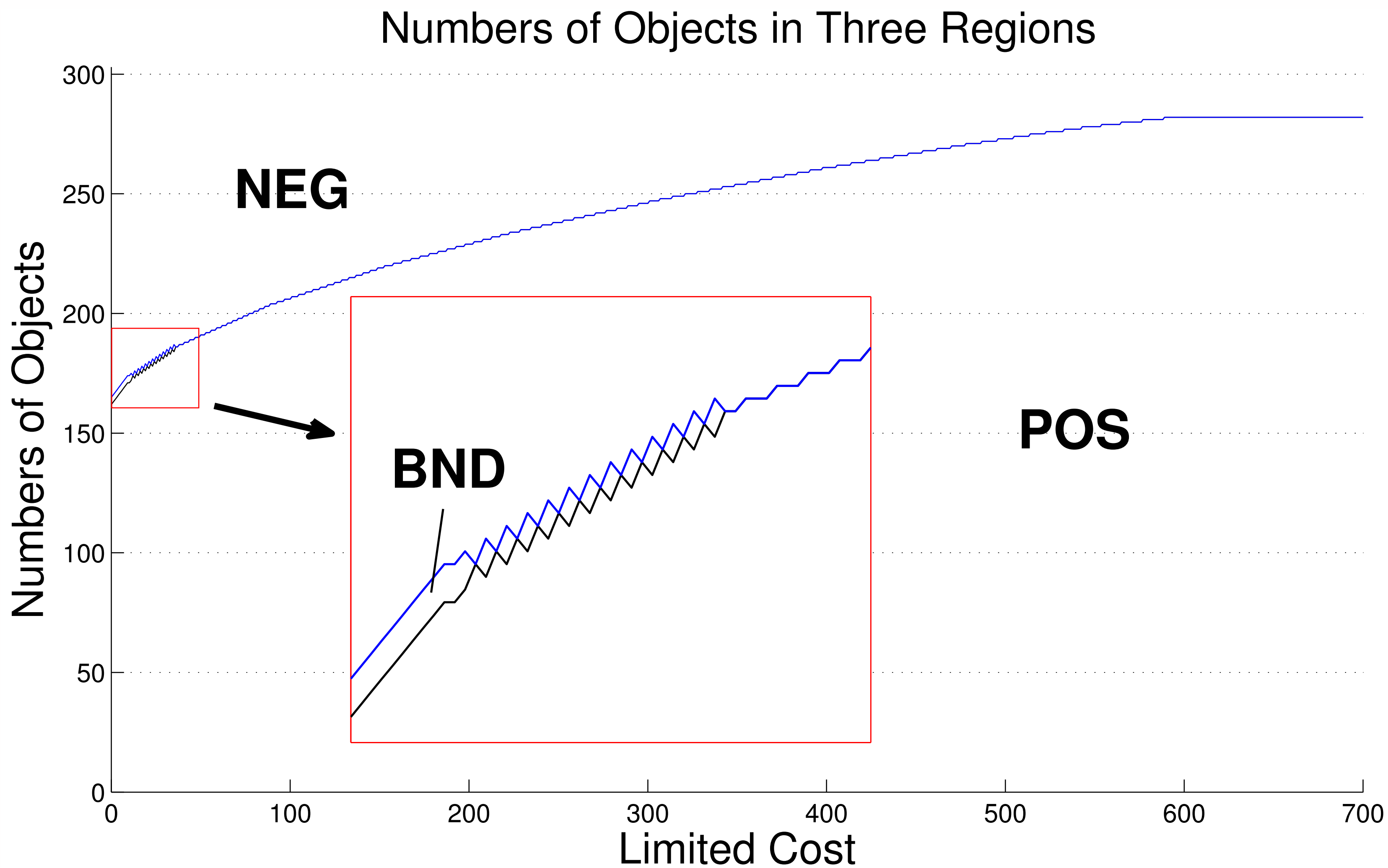
Experimental Results (cont.)

- Number of objects transferred under different cost



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Experimental Results (cont.)

- Comparison before and after reductions (model (i) and (ii))

Table 7.3: Comparison before and after reductions on model (i) and model (ii).

Data set	$\bar{B}'$	$\bar{C}'$	$\bar{B}$	$\bar{C}$	$\lvert R\rvert$	AVPs	Rules	RRules	Actions	RActions	Is improved
Hayes-Roth	525	154	525	**137**	3	3	12	0	49	131	Yes
Heart Disease	711	589	**837**	**142**	11	4.87	97	43	135	9876	Yes
Breast Cancer	138	374	**1446**	**576**	4	2.22	51	56	238	11900	Yes
Acute	540	241	540	**109**	2	2	1	0	11	0	Yes
CMC	5414	1988	**5492**	**1178**	9	4.16	245	154	541	36548	Yes
Haberman	142.02	42	**178.13**	**49**	3	2.05	35	2	12	1	Yes
Shuttle	18132	280545	18132	**8152**	4	1.92	686	4096	3022	2070070	Yes
TAE	608	494	608	**165**	5	2.38	23	3	60	600	Yes
Car	9978	6168	9978	6168	6	5.38	35	30	1663	56542	No

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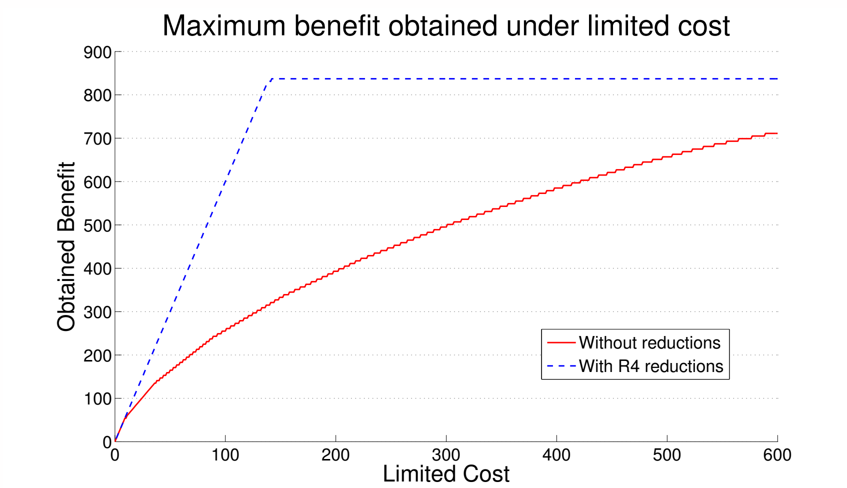
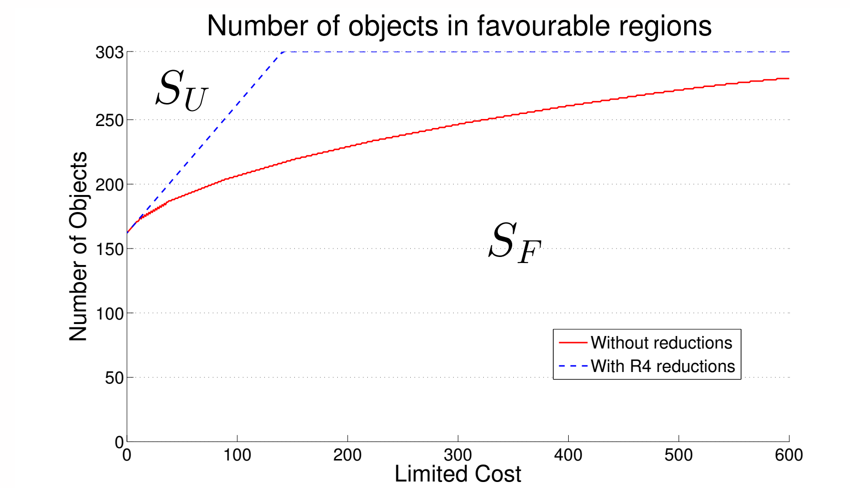
Experimental Results (cont.)

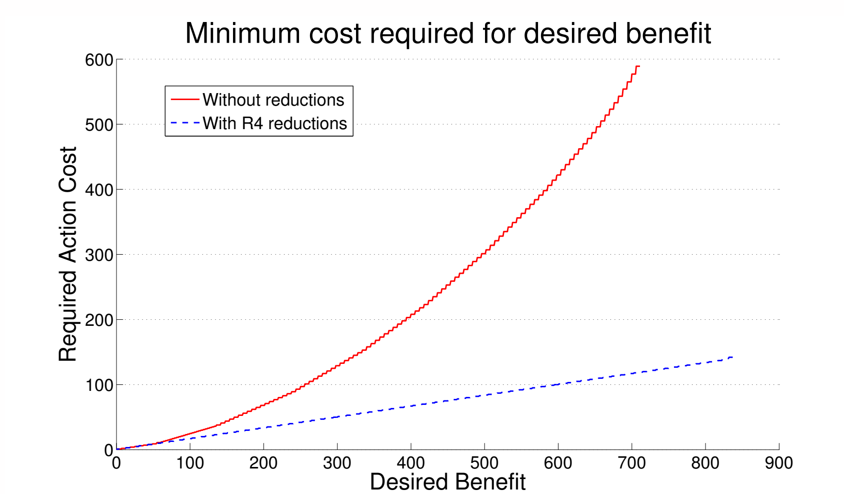
- Comparison before and after reductions (model (iii) and (iv))



(a) Results of model (iii).



(b) Numbers of transferred objects.

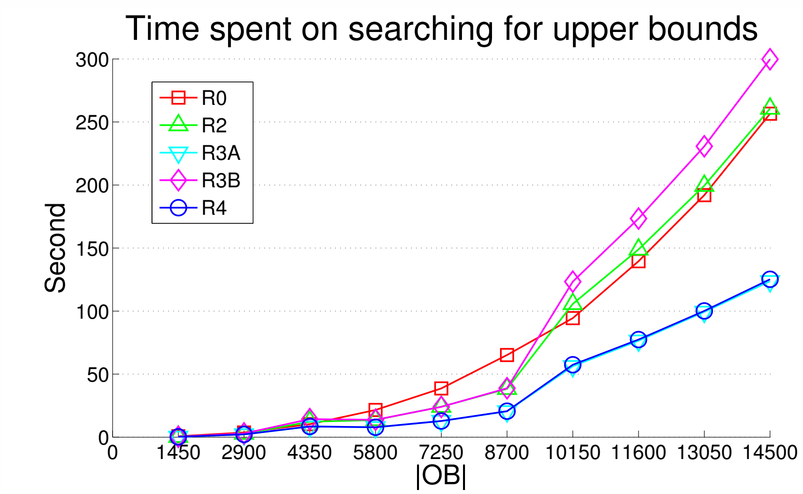


(c) Results of model (iv).

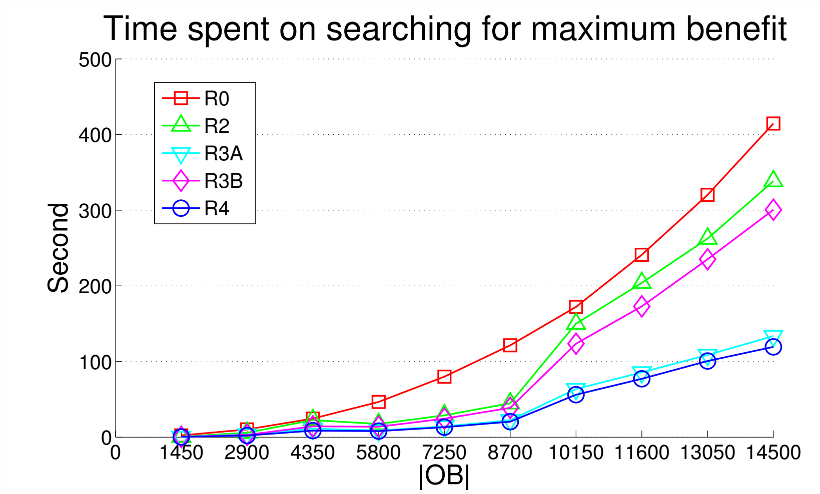
Figure 7.3: Experiments on actionable models (iii) and (iv) on the Heart Disease data set.

Experimental Results (cont.)

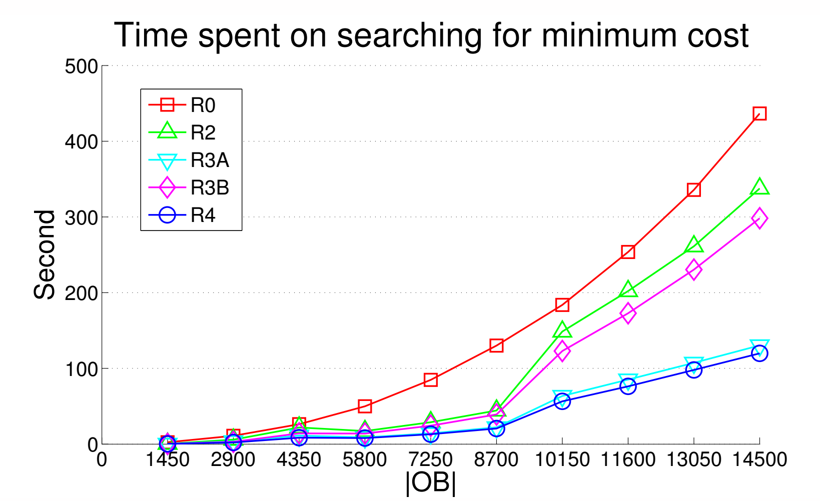
- Comparison on computation time



(a) Results on model (i) and model (ii).



(b) Results on model (iii).



(c) Results on model (iv).

Figure 7.5: Time spent on four actionable models on different sizes of the Shuttle data set.

Experimental Results (cont.)

- Comparison on different reduction methods

Table 7.4: Comparison between different methods on model (i) and model (ii).

#	Method	Hayes-Roth	Heart Disease	Breast Cancer	Acute	CMC	Haberman	Shuttle	TAE	Car
1	AA	525, 137	837, 142	1446, 576	540, 109	5492, 1178	178.13, 49	18132, 8152	608, 165	9978, 6168
2	AAd	525, 137	837, 154	1446, 742	540, 109	5492, 1196	171.02, 48	18132, 9719	608, 179	9978, 6168
3	AD	525, 137	837, 178	1446, 868	540, 109	5492, 1416	142.02, 42	18132, 10970	608, 217	9978, 6168
4	AdA	525, 137	837, 142	1446, 576	540, 109	5492, 1178	178.13, 49	18132, 11055	608, 165	9978, 6168
5	AdAd	525, 137	837, 154	1446, 742	540, 109	5492, 1196	171.02, 48	18132, 12059	608, 179	9978, 6168
6	AdD	525, 137	837, 178	1446, 868	540, 109	5492, 1416	142.02, 42	18132, 12271	608, 217	9978, 6168
7	DA	525, 137	837, 142	1446, 576	540, 109	5492, 1178	178.13, 49	18132, 11055	608, 165	9978, 6168
8	DAd	525, 137	837, 154	1446, 742	540, 109	5492, 1196	171.02, 48	18132, 12059	608, 179	9978, 6168
9	DD	525, 137	837, 178	1446, 868	540, 109	5492, 1416	142.02, 42	18132, 12271	608, 217	9978, 6168
10	LEM2	525, 137	837, 162	1446, 815	540, 139	5492, 1318	141.77, 43	18132, 21442	608, 403	9978, 6168

Conclusions

- An A3WD framework
  - Two statistical interpretations
  - One χ^2 based method for determining thresholds
  - Four actionable models
  - Four actionable rule mining algorithms
- A four-step reductions framework (R4)
  - An Addition strategy algorithm schema
  - A specific algorithm for attribute reduction and rule simplification

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Future Research Topics

- Correlation between actions and between sub-actions.
- Adapting decision tree for generating more general classification rules, hence more general action.
- Handling continuous attribute values for actionable rules.
- Adapting the A3WD to a sequential and dynamic scenario.
- Adapting the R4 framework to multi-objective problems.
- Applying utility theory to the actionable models (working).

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