The STP Model for Solving Imprecise Problems

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***Abstract*— Researchers have been attracted for years to studies on solving imprecise problems. The first step for solving an imprecise problem is to clarify the problem itself. However, in many of cases, the impreciseness of a problem is due to its own nature and often leaves them unsolvable. There are at least two reasons for the impreciseness and unclearness of a problem. The first reason is that there may not be a suitable language to present the problem in an understandable and clear way. The second reason is that the problem itself is not well-definable. This is quite similar to a research question that a researcher is trying to specify. A problem may only be fully understood and specified after all solutions are available. In this paper, we introduce a problem solving approach by searching possible solutions to an imprecise problem. This is an approach to specify and solve an imprecise problem by matching the problem with its solutions. We present a Solution-To-Problem (STP) model as a new approach for imprecise problem solving. Basic notions, measures and algorithms for such a problem solving process are studied.**

I. Introduction

Problems and questions are often raised in imprecise ways. The impreciseness of a problem or question is somehow due to its own nature of unclearness, fuzzyness or ill-structure [1]. The first reason for this is that we may not be able to properly present the problem or question in an understandable manner. The second reason may be due to the reality that we may not know every aspect of the problem or question. To obtain a solution to a problem or an answer to a question remains a challenge. In fact, the solution to an imprecise problem and the process of solving the problem are not unique. Jonassen studied the nature of imprecise and ill-structured problems and summarized that

Ill-structured problems possess multiple solutions, solution paths, fewer parameters which are less manipulable, and contain uncertainty about which concepts, rules, and principles are necessary for the solution or how they are organized and which solution is best [6].

A traditional way of solving an imprecise problem is to seek for a full understanding of the problem [4], [6], [10], [14]. The starting point of solving a problem is when the problem is clearly understood and identified [5]. However, when proper ways to clarifying or refining a problem are not available, the process of solving the problem and the effort to them may be extremely lengthy and costly. The consequence is that many problems are practically unsolvable. We argue that solving problems can be initiated before understanding them in full. In some cases, a problem may be more precisely presented if a possible solution set is available. The problem could be approximated by its solutions.

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The aim of this article is to introduce a novel problem solving approach, the solution-to-problem (STP) model. In fact, people use similar approaches to a certain extent. For example, keyword search is a popular method used by Web searching. The Web search engine's response to a query may stimulate the refinement of the query. Query formulation in information retrieval is one of theoretical background for such search mechanisms [15]. Another challenging scenario is the process of doing research. When a researcher could not formulate a research question precisely, some trial-and-error strategies would be explored in order to define the problem. The supervision of graduate students normally follows this pattern [2]. An imprecisely defined idea from a supervisor will eventually turn to a complete PhD dissertation. The STP approach is an effort in solving imprecise problems. It explores inaccurate, partially true solutions to achieve tractability of imprecise problems.

This paper is organized as follows. The STP approach is formalized in Section II. Section IV defines some basic measures used in problem solving especially for the STP approach. In Section III, we explain the STP model in the framework of granular computing. Section V studies the algorithms of problem solving with the STP process.

II. The STP Problem Solving Model

The STP model is formalized in this section. A general definition for problem solving is first given.

Definition 1: A problem solving space can be represented by the following triplet,

$$(\Omega, A, F),$$

where Ω represents the problem domain, A the solution domain, and F the solution function for the problems in Ω. In fact, $F(\omega)$ is a set of solutions to problem ω, i.e.

$$F : \Omega \rightarrow 2^A.$$

It is assumed that for any solution $a \in A$, there is a problem $\omega_a \in \Omega$ such that the solution a is in $F(\omega_a)$ and satisfies the problem ω. For any problem $\omega \in \Omega$, the task of problem-solving is to find out a solution in A that satisfies the problem ω to some extent.

A traditional way to solving any problem $\omega_0 \in \Omega$ consists of one or more steps. At each step, a refinement of the problem ω_0 is searched. For example, if the problem ω_0 itself can be fully understood, we search the solution domain A to solve the problem ω_0. Otherwise, problem $\omega_1 \in \Omega$ is searched such that the problem ω_1 is a refinement of the problem ω_0. If the problem ω_1 is fully understood and solved, the problem ω_0 is thought to be solved. Otherwise, we enter next step to find another refinement $\omega_2 \in \Omega$ of the problem ω_0, and so on.

This paper assumes that any problem can be solved within n $(n \geq 1)$ steps. The process of solving the problem ω_0 can be represented by trying to solve a sequence of problems:

$$< \omega_0, \ldots, \omega_n >,$$

where $\omega_i \in \Omega$ $(0 < i \leq n)$ represents a refinement of the problem ω_0.

The STP approach presents another way to solving the imprecise problem ω_0. The STP approach also consists of some steps. But at each step, a tentative solution is searched before pursuing more exact meaning of the problem ω_0. The tentative solution is viewed as an approximation to the nature of the problem ω_0. Since the solution is based on our incomplete understanding to the problem ω_0, it may not or only partially satisfy the problem ω_0.

The level that the solution a matches the problem ω_0 is defined as a real number t in the range of $[0, 1]$:

$$t : A \times \Omega \to [0, 1]. \tag{1}$$

The measure t is called the solution relevance measure. If a solution $a_0 \in A$ results in a solution relevance measure $t(a_0, \omega_0) > 0$, more information about the problem ω_0 may have been revealed. We can search the next solution $a_1 \in A$ based on the solution a_0 such that the solution a_1 is at least as preferable as the solution a_0, i.e. $t(a_1, \omega_0) \geq t(a_0, \omega_0)$. The above steps will be iterated until we can give a certain conclusion for the problem ω_0.

Definition 2: A model of the STP approach is defined as a pair:

$$(\alpha, S_{\omega_0}), \tag{2}$$

where α is a solution approximation function:

$$\alpha : \Omega \times A \to A, \tag{3}$$

S_{ω_0} a sequence of solutions:

$$S_{\omega_0} =< a_0, \ldots, a_n > \quad (a_i \in A,\ 0 \leq i \leq n). \tag{4}$$

The sequence S_{ω_0} or an STP process satisfies the following condition:

$$0 < t(a_0, \omega_0) \leq t(a_1, \omega_0) \leq \ldots \leq t(a_n, \omega_0). \tag{5}$$

It is aimed to find latter solutions that have a high level match to the problem ω_0. A solution a_i $(1 \leq i \leq n)$ is derived from the problem ω_0 and the previous solution a_{i-1}, i.e. $a_i = \alpha(\omega_0, a_{i-1})$.

Since we may find a set of solutions to the problem ω_0 at a step of the STP process, Formulas (3) and (4) in Definition 2 can be generalized as follows:

$$\alpha : \Omega \times 2^A \to 2^A, \tag{6}$$

$$S_{\omega_0} =< A_0, \ldots, A_n > . \tag{7}$$

Formula (7) indicates that S_{ω_0} is a sequence of solution sets. Formula (6) implies that $A_i = \alpha(\omega_0, A_{i-1})$ $(1 \leq i \leq n)$ or the solution set A_i is derived from the problem ω_0 and the solution set A_{i-1}.

In many cases, a solution set A_i is derived from the problem ω_0 and a special solution in A_{i-1}. Therefore, a choice function,

$$f : 2^A \to A, \tag{8}$$

should be defined. Formula (8) implies that a solution is chosen from the solution set A_{i-1} and is used as a reference to form the solution set A_i for the sake of simplicity. At each step i of the STP process, the solution chosen from the solution set A_i is denoted as a_i and $a_i = f(A_i)$. Equivalently, a variation of Formula (6) is defined as,

$$\alpha : \Omega \times A \to 2^A. \tag{9}$$

Without losing the generality of this paper, we assume that the STP model consists of Formulas (2), (7), (8), and (9). In addition, we often use $\alpha(a_{i-1})$ to represent the solution set A_i at the step i. The STP model has the following properties:

$$\forall a \in A_i,\ t(a, \omega_0) > 0 \quad (0 \leq i \leq n),$$

$$0 < t(a_{i-1}, \omega_0) \leq t(a_i, \omega_0) \quad (1 \leq i \leq n),$$

where $a_{i-1} = f(A_{i-1})$ and $a_i = f(A_i)$.

In this section, the STP model is characterized by a solution approximation function α and an approximation process S_{ω_0}. We define a solution relevance measure to evaluate the level that a solution that satisfies the problem ω_0. In each step of the STP process, the solution approximation function α provides a set of solutions. The solution set or one of its members plays a role of orientation in searching more preferable solutions in the next step.

III. Granular Computing Views of Imprecise Problems Solving

Granular computing is a structured way of thinking, analyzing, understanding, representing, and solving problems [8], [16], [18]. There are three basic ingredients of granular computing, namely, granules, levels, and hierarchies. A granule represents the abstraction of the real world. Granules may have different formats and meanings in different contexts. There may be multiple levels of granularity. A hierarchy connects levels together through a partial order. A top-down hierarchy can be implemented by adding more details in a step-wise manner. A bottom-up hierarchy can be implemented by ignoring irrelevant details in a step-wise manner [16].

The STP approach could also be viewed as a structured way of analyzing, understanding, representing, and solving problems. Usually, real world problems can be described at different levels: from exact and detailed to inexact and general. If a description of a problem can not satisfy the requirement of solving the problem, we may try to solve the problem at a coarse or inaccurate level. The approximate results may disclose some facts so that we can enter a more inexact level to understand the problem. Therefore, the STP process can be viewed as a top-down refinement process. The solution set at each step of the STP process can be viewed as a granule. In this granule, solutions are drawn together by their solution relevance to the problem. The more exactly a problem is described, the smaller the granule of approximate solutions to the problem.

The STP model is a framework for solving imprecise problems. It implies that solving an imprecise problem is actually a process of approximating the nature of the problem. Each step of the approximation process is to search possible solutions in a subset of the solution domain A. At each step, one may have to make decisions in selecting one of the solutions from the solution set. The solution may serve as the start point in searching for more preferable solutions. Various searching strategies and decision-making strategies, such as game theory and tabu search, may be adopted to form different STP models.

IV. Imprecise Problem Solving Measures

We need to evaluate how a solution satisfies the problem once the STP model has been established. It is important to identify procedures and methods to locate the solution space at each step of the STP process. We may also need to evaluate the relationships between steps of STP process.

In Section II, we argue that any solution of the solution space A can be considered as a solution to any problem in the problem space Ω. The solution relevance measure evaluates the viability of a solution given the problem ω_0 shown in Formula (1). The value $t(a, \omega_0)$ represents the level to which a solution a satisfies the problem ω_0. There are two extreme cases. The first extreme, $t(a, \omega_0) = 1$, indicates that solution a satisfies the problem ω_0. Another extreme, $t(a, \omega_0) = 0$, indicates that the solution a does not satisfy the problem ω_0 at all.

In the STP process, the solution domain A is only partially known before the problem ω_0 is absolutely understood and all solutions for the problem ω_0 have been found. Therefore, it may not be appropriate to search the whole solution domain A in all STP processes. Instead, we may only search possible solutions from a subset of the solution domain A. The subset is called the potential solution neighborhood of the problem ω_0 at step i (of the STP process) and is denoted as $PNS_i(\omega_0)$.

Each solution in $PNS_i(\omega_0)$ is considered to be a potential solution to the problem ω_0 according to a measure at the step i. The measure determining $PNS_i(\omega_0)$ is called the potential solution measure:

$$p : A \times \Omega \to [0, 1]. \tag{10}$$

The value $p(a, \omega_0)$ represents the possibility that the solution a falls into the potential solution neighborhood of the problem ω_0 at a step of the STP process. In particular, the condition $p(a, \omega_0) = 1$ indicates that the solution a is in the potential solution neighborhood; the condition $p(a, \omega_0) = 0$ indicates that the solution a is not in the potential solution neighborhood.

Since the potential solution neighborhood of the problem ω_0 may vary with the progression of solving the problem ω_0, the potential solution measure varies at different steps of the STP process, whereas the evaluation on the viability of a solution given the problem ω_0 is assumed to be consistent in the STP process. For any solution $a \in A$, $p_{i+1}(a, \omega_0)$ may not equal to $p_i(a, \omega_0)$, where $p_{i+1}(a, \omega_0)$ is the potential solution measure at step $i + 1$ and $p_i(a, \omega_0)$ the potential solution measure at the step i. The solution relevance measure and the potential solution measure are assumed to satisfy the following condition:

$$t(a', \omega_0) \geq t(a'', \omega_0) \Rightarrow p_i(a', \omega_0) \geq p_i(a'', \omega_0). \tag{11}$$

With the potential solution measure, the potential solution neighborhood of the problem ω_0 at step i is estimated by the following formula:

$$PNS_i(\omega_0) = \{a \in A \mid p_i(a, \omega_0) > 0\}. \tag{12}$$

According to the notion of the potential solution neighborhood of the problem, we can get the following formulas:

$$a \in \alpha(a_{i-1}, \omega_0) \Rightarrow p_i(a, \omega_0) > 0,$$
$$a(a_{i-1}, \omega_0) \subseteq PNS_i(\omega_0).$$

If a solution a is in the solution set of step i, the probability of it belonging to the potential neighborhood of the problem ω_0 is 1. Any solution in the solution set of step i is a solution of the ith potential neighborhood.

The following formula defines the similarity measures over a solution domain A:

$$s : A \times A \to [0, 1], \tag{13}$$

where $s(a_1, a_2)$ represents the similarity between solution a_1 and solution a_2. The condition $s(a_1, a_2) = 1$ indicates that the solution a_1 is equivalent to the solution a_2. The condition $s(a_1, a_2) = 0$ indicates that the solution a_1 is totally different from the solution a_2.

In order to simplify the problem, we assume that if the solution a_1 is similar with the solution a_2 and the solution a_1 possibly satisfies the problem ω_0, the solution a_2 also possibly satisfies the problem ω_0:

$$s(a_1, a_2) > 0 \ and \ t(a_1, \omega_0) > 0 \Rightarrow t(a_2, \omega_0) > 0.$$

We also assume that similar solutions fall into the same potential solution neighborhood of the problem:

$$s(a_1, a_2) > 0 \ and \ a_1 \in PNS_i(\omega_0) \Rightarrow a_2 \in PNS_i(\omega_0).$$

By introducing the similarity measure of solutions, we may be able to describe the relationships between steps of the STP process. In same cases higher potential levels of many solutions do not guarantee higher similarity levels. For example, an $a_i = f(A_i)$ and an $a_i = f(A_i)$ are chosen because $t(a_{i+1}, \omega_0) \geq t(a_i, \omega_0)$ or $tp(a_{i+1}, \omega_0) \geq p(a_i, \omega_0)$. However, the similarity level of a_i and a_{i+1} may be very low. In the extreme situation, say, $s(a, a_i) = 0$. It may imply that we want to explore the problem ω_0 at step $i + 1$ in a way totally different from the one at step i.

In this section, the measures of the STP process were studied. The solution relevance measure describes how well a solution satisfies a problem. Each step of STP process corresponds to a local solution space or the potential solution neighborhood of the problem. The potential solution measure is introduced to construct the potential solution neighborhood. The similarity measure between problems represents the relationships between steps of the STP process. In practice, the size of the potential solution neighborhoods and the size of the solution sets of the problem can be restricted by thresholds, which may improve the performance of the STP process. A threshold can be defined as any real number that is greater than one, for instance 0.5.

V. An Algorithm for The STP Problem Solving Approaches

Input: the problem ω_0, the span of h steps.
Begin
$\theta_0 = EstimateScopeThreshold(\omega_0)$;
$\sigma_0 = EstimateSolutionThreshold(\omega_0)$;
$a_{h-1} = ResultOfSpan(\omega_0, \theta_0, \sigma_0, h)$;
Compute $E\theta_{h-1}$;
Compute $E\sigma_{h-1}$;
Let $m = 1$;
While the STP process is not over do
If $t(a_{m\times h-1}, \omega_0)$ is sufficient high Then
$\theta_{m\times h} = E\theta_{m\times h-1} - \Delta$;
$\sigma_{m\times h} = E\sigma_{m\times h-1} - \Delta$;
Else If $t(a_{m\times h-1}, \omega_0)$ is high Then
$\theta_{m\times h} = E\theta_{m\times h-1}$;
$\sigma_{m\times h} = E\sigma_{m\times h-1}$;
Else If $t(a_{m\times h-1}, \omega_0)$ is low and
$|SNS_{m\times h}(\omega_0)|$ is much smaller than
$|PNS_{m\times h}(\omega_0)|$ Then
$\sigma_{m\times h} = E\sigma_{m\times h-1} + \Delta$;
$\theta_{m\times h} = E\theta_{m\times h-1}$;
Else
$\sigma_{m\times h} = E\sigma_{m\times h-1} + \Delta$;
$\theta_{m\times h} = E\theta_{m\times h-1} + \Delta$;
$a_{(m+1)\times h-1} =$
$ResultOfSpan(\omega_0, \theta_{m\times h}, \sigma_{m\times h}, h)$;
Compute $E\theta_{(m+1)\times h-1}$;
Compute $E\sigma_{(m+1)\times h-1}$;
Let $m = m + 1$;
End While
End

Fig. 1. An STP algorithm with dynamically optimizing thresholds

We first analyze the efficiency of the STP process. Section IV introduces the potential solution measure and the solution relevance measure. A practical issue is to determine the size of a potential solution neighborhood. Another issue is to determine the size of a solution set. Searching and evaluating solutions at each step of the STP process are time consuming. Therefore, the sizes of the potential solution neighborhood and the solution set affect the efficiency of the STP process. When the size of a potential solution neighborhood grows, we spend more time on searching solutions for building the solution set. In consequence, the chances to find a solution that satisfies the problem ω_0 increases. In contrast, when the size of the potential solution neighborhood is reduced, we may spend less time on searching solutions. However, the chance of finding satisfied solutions also decreases. The same conclusion applies to the size of a solution set. A solution set of smaller size may lead to less searching time but a lower probability of finding a solution. However, a larger solution set may provide a higher probability for a solution with a longer time search.

Input: the problem ω_0, the scope threshold θ,
the solution threshold σ, the span of h steps.
Begin
While $h > 0$ do
Estimate $PNS_i(\omega_0)$ based on threshold θ;
Compute $\alpha(\omega_0, A_{i-1})$;
Estimate $A_i = SNS_i(\omega_0)$ based on threshold σ;
$a_i = f(A_i)$;
$\theta = \theta \pm$random offset;
$\sigma = \sigma \pm$random offset;
$h = h - 1$;
End While
End

Fig. 2. The algorithm of searching solutions within h steps

A feasible approach to resolve the above dilemma is to establish thresholds to restrict the size of the potential solution neighborhood and the size of the solution set. We redefine Formula (12) into the following way:

$$PNS_i(\omega_0) = \{a \in A \mid p_i(a, \omega_0) > \theta_i\}, \tag{14}$$

where $\theta_i \in [0, 1]$ $(0 \le i \le n)$ is the scope threshold at step i (of the STP process). The solution sets A_i can be redefined as,

$$SNS_i(\omega_0) = \{a \in A_i) \mid t(a, \omega_0) > \sigma_i\}, \tag{15}$$

where $\sigma_i \in [0, 1]$ $(0 \le i \le n)$ is the solution threshold at step i (of the STP process).

We propose a strategy to determine the scope threshold and the solution threshold. The basic idea is that the optimal thresholds could be learnt based on the previous experiences obtained in the STP process. Each step of the STP process is viewed as a state denoted as S_i. A problem solver chooses the scope threshold θ_i and the solution threshold σ_i at the state S_i $(0 \le i \le n)$. The action leads to the transition from the state S_i to the state S_{i+1}. The above state transition produces a reward r_i to the problem solver. The reward is a function of a_i and the time used in searching and evaluating solutions at the step i, i.e. $r_i = r(t(a_i, \omega_0), time)$. The problem solver should choose actions that tend to increase rewards.

The algorithms described in Fig. 1 and Fig. 2 constitute an algorithmic description of the STP process. The algorithm described in Fig. 1 aims to optimize the scope thresholds and the solution thresholds for the next h $(h \ge 1)$ steps of the STP process through examining the effectiveness of approximating the problem ω_0 in the previous h steps of the STP process. The function *ResultOfSpan* described in Fig. 2 searches solutions within a span of h steps. The function *ResultOfSpan* also describes the actions carried out at a step of the STP process.

At the first step of the STP process, we need to estimate the size of the potential solution neighborhood derived from θ_0 and the size of the solution set derived from σ_0. The next

$h-1$ steps of the STP process are based on the estimated scope threshold θ_0 and solution threshold σ_0. When the current state is $s_{m\times h}$ $(m \geq 1)$, we can get the following summaries from the previous h steps:

$$\begin{aligned} E\theta_{m\times h-1} &= Avg(\sum_{i=1}^{h} \theta_{m\times h-i}), \\ E\sigma_{m\times h-1} &= Avg(\sum_{i=1}^{h} \sigma_{m\times h-i}). \end{aligned}$$

$E\theta_{(m-1)\times h}$ is the average size of the scope thresholds used in the previous h steps. $E\sigma_{(m-1)\times h}$ is the average size of the solution thresholds used in the previous h steps.

By evaluating the result at the step $m \times h - 1$, we can estimate the scope threshold $\theta_{m\times h}$ and the solution threshold $\sigma_{m\times h}$ for the step $m \times h$. There are four conditions. Firstly, if the result $a_{m\times h-1}$ satisfies the problem ω_0 very well, it may mean that the nature of the problem ω_0 has been approached and the solution set at the step $m\times h$ can be located accurately. In this case, we can reduce the scope of searching solutions and the size of the solution set. Here, we use the symbol Δ to generally represent the increase or decrease amount. Δ is assumed to be a function of $t(a_{m\times h-1}, \omega_0)$. Secondly, if the result $a_{m\times h-1}$ satisfies the problem ω_0 well, it means that we may be on a correct way towards solving the problem ω_0 and we should keep the exploration factors intact. The last two conditions are the situations that the result $a_{m\times h-1}$ cannot satisfy the problem ω_0. Thirdly, if the solution sets are too small during the previous h steps, the solution set should be enlarged at the next h steps. Lastly, if both the solution sets and the potential solution neighborhoods are not sufficiently large, they should be enlarged at the next h steps. Within the span of h steps, the size of the solution set and the size of the potential solution neighborhood at each step may be adjusted randomly based on $\sigma_{m\times h}$ and $\theta_{m\times h}$.

VI. Conclusions

In this paper, we introduce the STP model for imprecise problem solving. The impreciseness and unclearness of a problem derive from two different situations: First, there may not be a clear way to present a problem in a understandable way; Second, the problem itself may not be well-definable. A traditional way to solve an imprecise problem normally needs to articulate the problem first. However, the problem may only be fully understood and specified before all solutions are available. The STP problem solving approach generates solutions for an imprecise problem rather than defining the problem itself. A body of knowledge about the problem may be built through evaluating those solutions.

The STP model is a structural way of analyzing, understanding, representing, and solving problems. When a problem cannot be solved at explicit level, the STP approach tries to solve the problem at a coarse or inaccurate level. The approximate solutions or results may provide a chance for us to obtain a more exact level to understand the problem. Therefore, an STP process can be viewed as a top-down refinement process, which is under the umbrella of granular computing.

The STP model is characterized by a solution approximation function and the STP process. In each step of the STP process, the solution approximation function provides a set of solutions. A solution or a set of solutions of the solution set directs the way of searching more preferable solutions in the next step.

Three measures of the STP process are identified. The solution relevance measure evaluates the level that a solution satisfies the problem. The potential solution measure is used to estimate the scope of searching solutions at each step of the STP process. The similarity measure of solutions is used to describe the relationships between steps of the STP process.

We also present an algorithm of the STP process. The scope of searching solutions and the size of the solution set are optimized through trial-and-error interactions with the STP process.

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