12. Preference Reasoning

- The CSP framework is not expressive enough for many real world constraint problems.
  - Some problems are over constrained. Which constraints to relax?
  - Instead of looking for one solution (decision problem) we might need to:
    * find the best or a good solution (optimization problem)
    * discriminate between solutions

- We should distinguish between:
  - hard constraints: requirements that cannot be violated
  - soft constraints: desired properties that should be satisfied if possible
Quantitative vs Qualitative Preferences

- Quantitative (Cardinal) Preferences
  - My preference for soccer is 0.8, and for football is 0.6
  - Soft constraints: C-semiring
  - Pros: Easy to compare solutions
  - Cons: Difficult to find optimal solution and to deal with conditional preferences ("If C holds, I prefer A to B").

- Qualitative (Ordinal) Preferences: captures only ranking and not strength of preferences
  - Soccer > Football
  - CP Nets.
  - Pros: Easier to find optimal solution and to represent conditional preferences
  - Cons: Hard to compare solutions
Quantitative vs Qualitative Preferences

- A quantitative preference can be represented by a utility (or valuation) function

\[ u : X \rightarrow V \quad \text{where} \quad V \in \mathbb{N} \text{ or } \mathbb{R} \]

- A qualitative preference is a binary relation \( \preceq \) on \( X \).

- A qualitative preference can be represented by a utility function

\[ x \preceq y \quad \text{iff} \quad u(x) \leq u(y) \]
Quantitative Preferences: Soft Constraints

Generalize CSPs by having several levels of satisfiability (instead of satisfied / violated).

- **Fuzzy CSPs**
  - discretized preferences between 0 and 1.
  - quality of a solution is the minimum preference associated to constraints for that solution (pessimistic approach).

- **Lexicographic CSPs**
  - 2 solutions with the same minimum preference can be discriminated.
  - we compare lexicographically the ordered sequence of all the preferences given by the constraints to those 2 solutions.

- **Probabilistic CSPs**
  - each constraint has a probability of being present in the real world
  - find the solution with the highest probability (most robust solution)

- **Weighted CSPs. Mainly for over constrained problems.**
  - Each constraint is associated with a weight
  - find the solution with the max sum of weights
  - MaxCSP is a particular case where the weights are 0 or 1.
C-semiring

- Generalization of all previous formalisms: semiring-based soft constraint problems (SCSPs).

- Set $V$ ($V \in \mathbb{N}$ or $V \in \mathbb{R}$) with two operations:
  - Set values are those given to variables assignment. 1 and 0 for best and worst.
  - Comparison operator: $\oplus$
  - Combination operator: $\otimes$

- Ordering relation can be partial or total.
C-semiring

• Combination operator: $\otimes$
  - commutative, associative, distributes over $\oplus$, 1 is unit element

• Comparison operator: $\oplus$
  - commutative, associative, idempotent, 0 is unit element, 1 is absorbing element
  - A is better than B iff $A \oplus B = A$
Valued CSPs

• Like SCSP it is a generalization of all previous formalisms.

• Ordered monoid: ordered set $V$ with 1 combination operator $\otimes$.

• Ordering relation is total.
Values CSP instances

- Classical CSP: $V = \{true, false\} \quad \otimes = \land$
- Fuzzy CSP: $V = [0, 1] \quad \otimes = \text{max}$
- Weighted CSP: $V = [0, +\infty] \cap \mathbb{N} \quad \otimes = +$
- Probabilistic CSP (and Bayesian Net): $V = [0, 1] \quad \otimes = \ast$
• $V = \text{true, false}$. Worst value = false, best value = true

• $\oplus = \bigvee : A$ is better than $B$ iff $A \lor B = A$

• $\otimes = \bigwedge$
C-semiring instances: Weighted CSP

- $V = [0, +\infty] \cap \mathbb{N}$. Worst value=0, best value=$+\infty$
- $\oplus = \max$: A is better than B iff $\max(A, B)=A$
- $\otimes = +$
C-semiring instances: Weighted CSP

Unary constraints
value(SideDish=rice) = 0
value(SideDish=fries) = 1
value(Main=meat) = 2
value(Main=fish) = 0

Binary constraint
value(Main=meat & SideDish=rice) = 0
value(Main=meat & SideDish=fries) = 4
value(Main=fish & SideDish=rice) = 6
value(Main=fish & SideDish=fries) = 0
C-semiring instances: Weighted CSP

Four possible solutions
Main=fish, SideDish=rice
0 (Main=fish) +
0 (SideDish=rice) +
6 (Main=fish & SideDish=rice) = 6
Main=meat, SideDish=rice
2+0+0=2
Main=fish, SideDish=fries
0+1+0=1
Main=meat, SideDish=fries
2+1+4=7

Best solution (max):
Main=meat, SideDish=fries (7)
C-semiring instances: Fuzzy CSP

- $V = [0, 1]$. Best value = 1, worst value = 0
- $\oplus = \text{max}: A \text{ is better than } B \iff \max(A, B) = A$
- $\otimes = \text{min}$
**Unary constraints**

- value(SideDish=rice) = 0
- value(SideDish=fries) = 0.1
- value(Main=meat) = 0.2
- value(Main=fish) = 0

**Binary constraint**

- value(Main=meat & SideDish=rice) = 0
- value(Main=meat & SideDish=fries) = 0.4
- value(Main=fish & SideDish=rice) = 0.6
- value(Main=fish & SideDish=fries) = 0
Four possible solutions
Main=fish, SideDish=rice
\[\min[0 \ (\text{Main=fish}) ,
\quad 0 \ (\text{SideDish=rice}) ,
\quad 0.6 \ (\text{Main=fish & SideDish=rice})] = 0\]
Main=meat, SideDish=rice
\[\min[0.2,0,0]= 0\]
Main=fish, SideDish=fries
\[\min[0,0.1,0]= 0\]
Main=meat, SideDish=fries
\[\min[0.2,0.1,0.4]= 0.4\]

Best solution (max) :
    Main=meat, SideDish=fries (0.4)
Figure 1: Fuzzy CSP representation.
Using the C-semiring (or valued CSP):

- two solutions can be compared (with the $\oplus$ operator in the case of C-semiring).

- Best solution: complete and consistent assignment with the optimal combined cost.

- Finding the best solution requires a branch and bound algorithm with exponential time cost.
Naive solving algorithm using the classical CSP

- Representation: CSP problem $P \cup C_k$
  - $C_k$ is the global criteria constraint: total cost $> k$

- Solving method:

  $k = n$
  
  While (P U Ck inconsistent)
  
    solve P U Ck
  
    $k = k - 1$
  
  Return $k$ (optimal value)
Qualitative Preferences: CP Nets

- *Ceteris paribus* with conditional preference. If C is true, all other things being equal, I prefer A to B.

\[ C : A > B \]
Qualitative Preferences: CP Nets

Unconditional
Main=meat > Main=fish

Conditional
Main=fish :
SideDish=rice > SideDish=fries
Main=meat :
SideDish=fries > SideDish=rice
Figure 2: CP-Nets representation.