Representing and Reasoning about Motion in a Two-Dimensional World

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Abstract

This paper presents a point based spatio-temporal first order logic for representing the qualitative and quantitative spatial temporal knowledge needed to reason about motion in a two-dimensional space. The knowledge of a simplified world, a two-dimensional street network with active traffic lights, is represented, and the reasoning problem of how a robot moves from one place to another in the world is formalized with the proposed logic.

1 Introduction

Some of the most common human activities are walking, route-finding and navigation, which involve dealing with the physical world – the three dimensional space we live in. For example, when getting up in the morning, one goes from the bedroom to the bathroom through the door; finds something in the right place in the kitchen to eat and drink; walks (or drives) along some existing routes to school; finds one's way to the classroom through the building.

One does all of these daily things without any conscious thought or calculation when in a familiar environment. What kind of knowledge does one have about the environment in which one lives? How does one use this knowledge in daily activities of walking, route-finding or navigation? Is it this kind of knowledge that assures us that we can reach our destination? Further, is this knowledge representable, learnable, and manipulatable? Can this kind of knowledge be stored in a robot to enable it to do the same thing? Exploring these questions leads us closer to an explicit understanding of the commonsense knowledge of space, which we consider important for understanding how people deal with space and also important for building robots capable of dealing with their environments.

This paper presents a point based spatio-temporal logic to represent the qualitative and quantitative spatial temporal knowledge needed to reason about motion in a two-dimensional space. In the next section we present the logic with detailed syntax and semantics. Then we examine the problem of how to get from one place to another in a two-dimensional street-network and formalize this problem in the proposed logic.

2 A spatio-temporal logic

The proposed spatio-temporal logic is based on a first order temporal logic called RGCH [2]. RGCH uses real valued functions to represent point based temporal information and these functions are integrated to represent interval based information. We extend RGCH's real valued functions to include a spatial pa-

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rameter so that it can be used to represent knowledge of both space and time.

2.1 Syntax

Given the following disjoint sets:

- TC: a set of temporal constant symbols;
- TV: a set of temporal variable symbols;
- OC: a set of orientational constant symbols;
- OV: a set of orientational variable symbols;
- LC: a set of locational constant symbols;
- LV: a set of locational variable symbols;
- UC: a set of non-spatio-temporal constant symbols;
- UV: a set of non-spatio-temporal variable symbols;
- UF: a set of non-spatio-temporal function symbols;
- F: a set of function symbols.

Terms are defined as follows:

- All members of TC and TV are temporal terms;
- If t_1 and t_2 are temporal terms, then $t_1 + t_2$, $t_1 - t_2$ and $t_1 \times t_2$ are temporal terms;
- All members of OC and OV are orientational terms;
- If d_1 and d_2 are orientational terms, then $add(d_1, d_2)$, $sub(d_1, d_2)$ and $opposite(d_1)$ are orientational terms;
- All members of LC and LV are locational terms;
- If p_1 and p_2 are locational terms, then $|p_1, p_2|$ is a locational term, and $orient(p_1, p_2)$ is an orientational term;
- All orientational and locational terms are spatial terms;
- All members of UC and UV are non-spatio-temporal terms;
- If $term_1, term_2, \ldots, term_n$ are non-spatio-temporal terms, and $f \in UF$ is an n_ary function, then $f(term_1, term_2, \ldots, term_n)$, is a non-spatio-temporal term;
- All temporal, spatial and non-spatio-temporal terms are terms;
- If $term_1, term_2, \ldots, term_n$ are non-spatio-temporal terms, t, t_1, t_2 are temporal terms, s, s_1, s_2 are spatial terms, and $f \in F$ is an (n+2)_ary function, then $f(term_1, term_2, \ldots, term_n, s, t)$, $\int_{t_1}^{t_2} f(term_1, term_2, \ldots, term_n, s, t)dt$, $\int_{s_1}^{s_2} f(term_1, term_2, \ldots, term_n, s, t)ds$ are terms.

The well-formed formulas (wffs) are defined as follows:

- If $term_1, term_2$ are terms, then $term_1 = term_2$ is a wff;
- If t_1, t_2 are temporal terms, then $t_1 < t_2$ is a wff;
- If d_1, d_2 are orientational terms, then $d_1 < d_2$ is a wff;
- If φ_1, φ_2 are wffs, then $\neg\varphi_1$, $\varphi_1 \wedge \varphi_2$ are wffs;
- If φ is a wff and v is a variable from TV, OV, LV or UV, then $\forall v(\varphi)$ is a wff.

We assume the usual definitions of $\vee$, $\Rightarrow$, $\exists$, $\exists!$, $\oplus$, $\leq$, $>$, $\geq$, etc.

2.2 Semantics

The real number set $\mathbb{R}$ is the semantic domain for temporal objects, $\mathbb{R} \times \mathbb{R}$ for locational objects, a subset $\mathbb{R}'$ of $\mathbb{R}$ $(-180, 180]$ for orientational objects and $\mathbb{U}$, a non-empty set of individuals, is the semantic domain for the remaining objects.

An interpretation is a tuple $I = \langle \mathbb{R}, \mathbb{U}, \alpha \rangle$ where α is an interpretation function that maps each (n+2)_ary function in F to an (n+3)_ary function from $\mathbb{U}^n \times \mathbb{R}^2 \times \mathbb{R}$ to $\mathbb{R}$ if it contains a locational argument, or to an (n+2)_ary function from $\mathbb{U}^n \times \mathbb{R}' \times \mathbb{R}$ to $\mathbb{R}$ if it contains an orientational argument, and maps each n_ary function in UF to an n_ary function from $\mathbb{U}^n$ to $\mathbb{U}$.

A variable assignment β is a function that maps each temporal variable to an element of $\mathbb{R}$, each locational variable to an element of $\mathbb{R}^2$, each orientational variable to an element of $\mathbb{R}'$ and each other variable to an element of $\mathbb{U}$.

The meaning of terms is defined as follows:

- For a constant $c \in TC$ (LC, OC or UC), $I(c) = \alpha(c)$;
- For a variable $v \in TV$ (LV, OV or UV), $I(v) = \beta(v)$;
- For an n_ary function $f \in F$ (or UF) and terms $term_1, term_2, \ldots, term_n$,

  $I(f(term_1, term_2, \ldots, term_n)) =$
  $\alpha(f)(I(term_1), I(term_2), \ldots, I(term_n))$;
- For an (n+2)_ary function $f \in F$ and terms $term_1, \ldots, term_n$, s, s_1, s_2, t, t_1, t_2,

  $I(\int_{t_1}^{t_2} f(term_1, \ldots, term_n, s, t)dt) =$
  $\int_{I(t_1)}^{I(t_2)} \alpha(f)(I(term_1), \ldots, I(term_n), I(s), t)dt$,

  $I(\int_{s_1}^{s_2} f(term_1, \ldots, term_n, s, t)ds) =$
  $\int_{I(s_1)}^{I(s_2)} \alpha(f)(I(term_1), \ldots, I(term_n), s, I(t))ds$;
- If t_1, t_2 are temporal terms, then
  $I(t_1 + t_2) = I(t_1) + I(t_2)$,
  $I(t_1 - t_2) = I(t_1) - I(t_2)$,
  $I(t_1 \times t_2) = I(t_1) \times I(t_2)$;
- If d_1, d_2 are orientational terms, then

  $$I(add(d_1, d_2)) = \begin{cases} I(d_1) + I(d_2) & \text{if } -180 < I(d_1) + I(d_2) \leq 180, \\ I(d_1) + I(d_2) + 360 & \text{if } I(d_1) + I(d_2) < -180, \\ I(d_1) + I(d_2) - 360 & \text{if } 180 \leq I(d_1) + I(d_2), \end{cases}$$

  $$I(sub(d_1, d_2)) = \begin{cases} I(d_1) - I(d_2) & \text{if } -180 < I(d_1) - I(d_2) \leq 180, \\ I(d_1) - I(d_2) + 360 & \text{if } I(d_1) - I(d_2) < -180, \\ I(d_1) - I(d_2) - 360 & \text{if } 180 \leq I(d_1) - I(d_2), \end{cases}$$

  $I(opposite(d_1)) = I(sub(d_1, 180))$;
- If p_1 and p_2 are locational terms, let $I(p_1) =< x_{p_1}, y_{p_1} >$ and $I(p_2) =< x_{p_2}, y_{p_2} >$, then

  $I(|p_1, p_2|) = \sqrt{(x_{p_2} - x_{p_1})^2 + (y_{p_2} - y_{p_1})^2}$,
  $I(orient(p_1, p_2)) = \arctan((y_{p_2} - y_{p_1})/(x_{p_2} - x_{p_1}))$.

The interpretation I and the variable assignment β satisfy a wff φ (written $I \models \varphi[\beta]$) under the conditions:

- $I \models term_1 = term_2[\beta]$ iff $I(term_1) = I(term_2)$;
- $I \models term_1 < term_2[\beta]$ iff $I(term_1) < I(term_2)$;
- $I \models \neg\varphi[\beta]$ iff $I \not\models \varphi[\beta]$;
- $I \models (\varphi_1 \wedge \varphi_2)[\beta]$ iff $I \models \varphi_1[\beta]$ and $I \models \varphi_2[\beta]$;
- $I \models (\forall v(\varphi))[\beta]$ iff

  $$I \models \varphi[\beta_{d/v}] \begin{cases} \text{for all } d \in \mathbb{R} & \text{if } v \in TV, \\ \text{for all } d \in \mathbb{R}^2 & \text{if } v \in LV, \\ \text{for all } d \in \mathbb{R}' & \text{if } v \in OV, \\ \text{for all } d \in \mathbb{U} & \text{otherwise.} \end{cases}$$

3 Motion in a two-dimensional street network

We consider the problem of how to get from one place to another in a simple world. The world consists of a two-dimensional street network with traffic lights along streets changing their color periodically, and a robot which moves on the street. We first describe the structure of the street network, how the traffic lights may change, the states of the robot in the space, and the actions this robot can take under different situations. Then we prove that it is possible for a robot to go from one place to another as long as it knows the existing routes and follows the traffic rules.

3.1 The street network

A street network consists of streets which intersect, and of places on the street (Figure 1). In the representation of space, places (abstracted as spatial points in the space's coordinate system) are a central notion, and streets (abstracted as straight lines, called paths, which have length $|p_1, p_2|$ and direction $orient(p_1, p_2)$) are considered to consist of ordered infinite places and are marked by their two end points.

We introduce relations on places and paths:

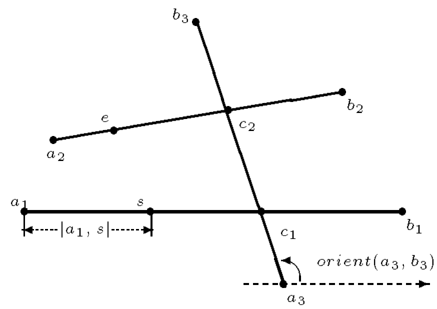

Figure 1: A simple street network

Definition 3.1 *Let* (a, b) *be a path and* p *a place other than* a *or* b*. The place* p *is on the path* (a, b) *if and only if path* (a, p) *and path* (p, b) *have the same orientation:*

$on(p,(a,b))$ *iff* $orient(a,p) = orient(p,b)$.

Definition 3.2 *Let* (a, b) *be a path, and* d *the value of* orient(a, b)*. We define a binary relation* $\preceq_d$ *between two places on a path* (a, b) *with respect to orientation* d *as:*

$p_1 \preceq_d p_2$ *iff* $(|a,p_1| \leq |a,p_2|) \wedge (|b,p_2| \leq |b,p_1|)$.

If $p_1 \preceq_d p_2$, we say p_1 precedes p_2 with respect to the orientation d. Therefore, the places on a path are totally ordered with respect to the path's orientation. The relation $\preceq_d$ is sometimes written as $\preceq$ if no confusion arises. As usual, $p_1 \prec p_2$ is defined as $p_1 \preceq p_2 \wedge p_1 \neq p_2$, and $a \preceq p \preceq b$ is an abbreviation for $a \preceq p \wedge p \preceq b$.

Definition 3.3 *Two paths* meet *if and only if the endpoint of the first path is the starting point of the second path and their orientations are not opposites:*

$meet((a_1,b_1),(a_2,b_2))$ *iff*

$(a_2 = b_1) \wedge orient(a_2,b_2) \neq opposite(orient(a_1,b_1))$.

Definition 3.4 *Two paths* cross *each other if and only if they have a unique common place:*

$cross((a_1,b_1),(a_2,b_2))$ *iff*

$\exists! p(a_1 \prec p \prec b_1 \wedge a_2 \prec p \prec b_2)$.

Given two places in a street network, there may exist some paths connecting them. We call such a sequence of meeting paths a *route* and assume that circuits are not allowed (i.e., no place appears twice in a route). Thus the linear order relation $\prec$ can be extended to paths with respect to two given places.

Definition 3.5 *Let* $(\mathrm{a}_1, \mathrm{b}_1)$ *and* $(\mathrm{a}_2, \mathrm{b}_2)$ *be two paths in a street network. Path* $(\mathrm{a}_1, \mathrm{b}_1)$ *precedes path* $(\mathrm{a}_2, \mathrm{b}_2)$ *with respect to the route from* a_1 *to* b_2 *(written* $(\mathrm{a}_1, \mathrm{b}_1) \preceq_{[\mathrm{a}_1,\mathrm{b}_2]} (\mathrm{a}_2, \mathrm{b}_2)$*) if and only if path* $(\mathrm{a}_1, \mathrm{b}_1)$ *meets path* $(\mathrm{a}_2, \mathrm{b}_2)$*, or there is a path* (a, b) *such that path* $(\mathrm{a}_1, \mathrm{b}_1)$ *meets path* (a, b) *and path* (a, b) *precedes path* $(\mathrm{a}_2, \mathrm{b}_2)$ *with respect to the route from* a *to* b_2*:*

$(a_1,b_1) \preceq_{[a_1,b_2]} (a_2,b_2)$ *iff*

$meet((a_1,b_1),(a_2,b_2)) \vee$

$\exists a,b(meet((a_1,b_1),(a,b)) \wedge (a,b) \preceq_{[a,b_2]} (a_2,b_2))$.

Definition 3.6 *Let* s *and* e *be two places,* $(\mathrm{a}_1, \mathrm{b}_1)$ *and* $(\mathrm{a}_2, \mathrm{b}_2)$ *be two paths in a street network. Path* $(\mathrm{a}_1, \mathrm{b}_1)$ *precedes path* $(\mathrm{a}_2, \mathrm{b}_2)$ *in the route from* s *to* e *is defined as:*

$(a_1,b_1) \preceq_{[s,e]} (a_2,b_2)$ *iff*

$\exists s',e'((s,s') \preceq_{[s,e]} (e',e) \wedge$

$(s,s') \preceq_{[s,b_1]} (a_1,b_1) \preceq_{[a_1,b_2]} (a_2,b_2) \preceq_{[a_2,e]} (e',e))$.

3.2 Traffic lights

The street network does not change over time. However, traffic lights located at some places along the street periodically change their color between red and green, where a red light prevents traffic from moving. To represent the traffic lights in the streets, we introduce a function $light(p,t)$:

Definition 3.7 *The function* light(p, t) *has three values* $\{0, 1, 2\}$ *where:*

$$light(p,t) = \begin{cases} 0 & \textit{if} \text{ at time } t \text{ there is no traffic light at place } p, \\ 1 & \textit{if} \text{ at time } t \text{ the traffic light at place } p \text{ is red}, \\ 2 & \textit{if} \text{ at time } t \text{ the traffic light at place } p \text{ is green}. \end{cases}$$

The following axioms capture common rules of changing traffic lights:

Axiom 3.1 *If there is a traffic light at place* p*, then it will be either* red *or* green*; If there is no traffic light at* p*, then there is always no traffic light at* p*:*

$\forall p(\exists t'(light(p,t') \neq 0) \Rightarrow$
$\forall t(light(p,t) = 1 \oplus light(p,t) = 2))$,

$\forall p(\exists t'(light(p,t') = 0) \Rightarrow \forall t(light(p,t) = 0))$.

Axiom 3.2 *A traffic light alternates between* red *and* green *on a regular basis (e.g., every* T *time units):*

$\forall p, t(light(p,t) = 1 \Rightarrow light(p, t+T) = 2),$

$\forall p, t(light(p,t) = 2 \Rightarrow light(p, t+T) = 1).$

From these axioms, we infer the following theorem:

Theorem 3.1 *If over* (t_1, t_2) *a traffic light is* red (green) *then over* $(t_1 + T, t_2 + T)$ *it is* green (red)*:*

$$\forall p, t_1, t_2(\forall t'(t_1 < t' \leq t_2 \Rightarrow light(p,t') = 1) \Rightarrow \forall t''(t_1 + T < t'' \leq t_2 + T \Rightarrow light(p, t'') = 2)),$$

3.3 The robot

We assume there is only one agent, a robot called "R", which can move in two-dimensional space. To predict that R will reach its destination if it plans to go from a source place to a goal place, we require its position in the space at different times, the actions it can perform under different situations, and the consequences of different actions.

The robot's state is determined by its location and direction, which are represented by the function $at(R,p,t)$ and $face(R,d,t)$.

Definition 3.8 *The function* at(R, p, t) *describes R's location:*

$$at(R,p,t) = \begin{cases} 1 & \text{if at time } t \ R \text{ is at place } p, \\ 0 & \text{if at time } t \ R \text{ is not at place } p. \end{cases}$$

Definition 3.9 *The function* face(R,d,t) *describes R's direction:*

$$face(R,d,t) = \begin{cases} 1 & \text{if at time } t \ R \text{ faces direction } d, \\ 0 & \text{if at time } t \ R \text{ does not face direction } d. \end{cases}$$

The actions that R is allowed to perform are *move* and *turn* which are described by the functions *move_velocity* and *turn_velocity*.

Definition 3.10 *The function move_velocity(R,d,t) equals the velocity at which R is moving in direction* d *at time* t.

$$move\_velocity(R,d,t) \begin{cases} > 0 & \text{if at time } t \ R \text{ is moving in the direction } d, \\ < 0 & \text{if at } t \ R \text{ is moving in the direction opposite } d, \\ = 0 & \text{if at time } t \ R \text{ is not moving.} \end{cases}$$

Definition 3.11 *The function turn_velocity(R,p,t) is the velocity at which R is turning at place* p *at time* t.

$$turn\_velocity(R,p,t) \begin{cases} > 0 & \text{if at time } t \ R \text{ is turning left at place } p, \\ < 0 & \text{if at time } t \ R \text{ is turning right at place } p, \\ = 0 & \text{if at time } t \ R \text{ is not turning at place } p. \end{cases}$$

The functions $move\_velocity(R,d,t)$ and $turn\_velocity(R,p,t)$ can be integrated over a time interval (t_1, t_2) to represent the length that R has moved and the angle that R has turned respectively.

Definition 3.12 *Let* (a, b) *be a path,* p_1, p_2 *two places on* (a, b)*,* d=orient(a, b)*, and* $at(R, p_1, t_1) = 1$*,* $at(R, p_2, t_2) = 1$*. We define:*

$$\int_{t_1}^{t_2} move\_velocity(R,d,t)dt = \begin{cases} |p_1, p_2| & \text{if } d = orient(p_1, p_2), \\ -|p_1, p_2| & \text{if } d = orient(p_2, p_1). \end{cases}$$

Definition 3.13
Let p *be a place,* $face(R, d_1, t_1) = 1$ *and* $face(R, d_2, t_2) = 1$*. We define:*

$$\int_{t_1}^{t_2} turn\_velocity(R,p,t)dt = sub(d_2, d_1).$$

When an agent moves, there is a change in its location but no change in its direction; when it turns, there is a change in its direction but not in its location. This is axiomatized as follows:

Axiom 3.3 *If R is at place* p_1 *at time* t_1*, and travels the path* (p_1, p_2) *over time interval* (t_1, t_2)*, then at time* t_2 *R is at place* p_2*:*

$$\forall t_1, t_2((at(R, p_1, t_1) = 1 \wedge \int_{t_1}^{t_2} move\_velocity(R, orient(p_1, p_2), t)dt = |p_1, p_2|) \Rightarrow at(R, p_2, t_2) = 1).$$

Axiom 3.4 *If R is facing direction* d_1 *at time* t_1 *and turns the angle* $sub(d_2, d_1)$ *over time interval* (t_1, t_2)*, then at time* t_2 *R is facing direction* d_2*:*

$$\forall t_1, t_2((face(R, d_1, t_1) = 1 \wedge \int_{t_1}^{t_2} turn\_velocity(R,p,t)dt = sub(d_2, d_1)) \Rightarrow face(R, d_2, t_2) = 1).$$

Axiom 3.5 *If R is facing direction* d *at time* t_1 *and moves over time interval* (t_1, t_2)*, then at time* t_2 *R is still facing direction* d*:*

$\forall d, t_1, t_2((face(R,d,t_1) = 1 \wedge$
$\forall t(t_1 < t < t_2 \Rightarrow move\_velocity(R,d,t) \neq 0))$
$\Rightarrow face(R,d,t_2) = 1).$

Axiom 3.6 *If R is at place* p *at time* t_1 *and turns over time interval* $(\mathrm{t}_1, \mathrm{t}_2)$*, then at time* t_2 *R is still at place* p*:*

$\forall p, t_1, t_2((at(R,p,t_1) = 1 \wedge$
$\forall t(t_1 < t < t_2 \Rightarrow turn\_velocity(R,p,t) \neq 0))$
$\Rightarrow at(R,p,t_2) = 1).$

3.4 Moving from one place to another

The particular problem we are interested in is that of a robot moving from place s to place e on a given route. The robot may encounter traffic lights along the way and must stop for red ones. We want to prove that the robot will eventually reach its destination. If a departure time and velocities are given, then we also want to approximate the arrival time. To solve this problem, we need axioms to capture how the robot applies its knowledge to guide its motion along streets.

Axiom 3.7 *If R is moving on path* (a, b) *towards* b *then R keeps moving until it encounters a red light or a corner for turning, or reaches* b*:*

$\forall a, b, p_0, t_0((a \preceq p_0 \prec b \wedge at(R,p_0,t_0) = 1 \wedge$
$move\_velocity(R, orient(a,b), t_0) > 0) \Rightarrow$
$\exists p_1, t_1(\forall t(t_0 < t < t_1 \Rightarrow$
$move\_velocity(R, orient(a,b), t) > 0) \wedge$
$on(p_1, (a,b)) \wedge at(R,p_1,t_1) = 1 \wedge$
$(move\_velocity(R, orient(a,b), t_1) = 0 \Leftrightarrow$
$((light(p_1,t_1){=}1 \wedge turn\_velocity(R,p_1,t_1){=}\ 0) \vee$
$(light(p_1,t_1){\neq}1 \wedge turn\_velocity(R,p_1,t_1){\neq}0) \vee$
$p_1 = b)))).$

Axiom 3.8 *If R is waiting for a red light at place* p_0 *and facing direction* d*, when the light turns green, R moves forward or turns:*

$\forall d, p_0, t_0((light(p_0,t_0) = 1 \wedge$
$at(R,p_0,t_0) = 1 \wedge face(R,d,t_0) = 1 \wedge$
$move\_velocity(R,d,t_0) = 0 \wedge$
$turn\_velocity(R,p_0,t_0) = 0) \Rightarrow$
$\exists t_1(\forall t(t_0{<}t{<}t_1 {\Rightarrow} (move\_velocity(R,d,t){=}0 \wedge$
$turn\_velocity(R,p_0,t) = 0)) \wedge$
$(move\_velocity(R,d,t_1) \neq 0 \vee$
$turn\_velocity(R,p_0,t_1) \neq 0) \wedge$
$light(p_0,t_1) = 2)).$

Axiom 3.9 *If R is moving on path* $(\mathrm{a}_1, \mathrm{b}_1)$ *towards a destination, and path* $(\mathrm{a}_1, \mathrm{b}_1)$ *meets and precedes path* $(\mathrm{a}_2, \mathrm{b}_2)$*, then R will turn when it reaches place* b_1 *and continue moving on path* $(\mathrm{a}_2, \mathrm{b}_2)$*:*

$\forall a_1, a_2, b_1, b_2((\exists p_0, t_0, t_b(p_0 \prec b_1 \wedge t_0 < t_b \wedge$
$at(R,p_0,t_0) = 1 \wedge$
$\int_{t_0}^{t_b} move\_velocity(R, orient(a_1,b_1), t)dt{=}|p_0, b_1|) \wedge$
$meet((a_1,b_1),(a_2,b_2)) \wedge (a_1,b_1) \preceq (a_2,b_2)) \Rightarrow$
$\exists t_b''(\int_{t_b}^{t_b''} turn\_velocity(R, b_1, t)dt =$
$sub(orient(a_2,b_2), orient(a_1,b_1)) \wedge$
$\forall d, t(t_b \leq t < t_b'' \Rightarrow$
$(at(R,b_1,t) = 1 \wedge move\_velocity(R,d,t) = 0)) \wedge$
$face(R, orient(a_2,b_2), t_b'') = 1 \wedge$
$move\_velocity(R, orient(a_2,b_2), t_b'') > 0)).$

To predict an approximate time t at which R reaches its destination, we introduce function $min(x_1, x_2, f(r_1, \ldots, r_i, x, r_{i+1}, \ldots, r_n))$ which has the minimum value of the function $f(r_1, \ldots, r_i, x, r_{i+1}, \ldots, r_n)$ when its parameter x varies over the range $[x_1, x_2]$.

Theorem 3.2 *If R is moving on a path* (a,b) *towards* b *without going backwards or stopping, then the time* t_b *when it reaches b is bounded as follows:*

$\forall a, b, p_0, t_0, t_b((a \preceq p_0 \prec b \wedge at(R,p_0,t_0) = 1 \wedge$
$move\_velocity(R, orient(a,b), t_0) > 0 \wedge$
$\forall t(t_0{<}t{<}t_b {\Rightarrow} move\_velocity(R, orient(a,b), t){>}0) \wedge$
$at(R,b,t_b) = 1) \Rightarrow$
$t_b - t_0 \leq |p_0, b|/$
$min(t_0, t_b, move\_velocity(R, orient(a,b), t))).$

Theorem 3.3 *If at time* $\mathrm{t_b}$ *R begins turning from a path* (a, b) *to a path* (b, c)*, then the time* t_b' *when it is on path* (b, c) *is bounded as follows:*

$\forall a, b, c, t_b, t_b'((face(R, orient(a,b), t_b) = 1 \wedge$
$turn\_velocity(R,b,t_b) \neq 0 \wedge$
$\forall t(t_b < t < t_b' \Rightarrow turn\_velocity(R,b,t) \neq 0) \wedge$
$face(R, orient(b,c), t_b') = 1) \Rightarrow$
$t_b' - t_b \leq sub(orient(b,c), orient(a,b))$
$/min(t_b, t_b', turn\_velocity(R,b,t))).$

3.5 Example

We conclude with an example where we prove that it is possible for the robot to get from one place to another.

A robot R is moving from place s to place e and there is a traffic light at both places c_1 and c_2 (Figure 1) which change color $(red, green)$ every T time units:

Given

1. Structure of streets:

   $cross((a_1,b_1),(a_3,b_3))$,
   $on(c_1,(a_1,b_1))$, $on(c_1,(a_3,b_3))$,
   $cross1((a_2,b_2),(a_3,b_3))$,
   $on(c_2,(a_2,b_2))$, $on(c_2,(a_3,b_3))$,
   $\forall p(on(p,(a_1,b_1)) \Rightarrow \neg on(p,(a_2,b_2)))$,
   $\forall p(on(p,(a_2,b_2)) \Rightarrow \neg on(p,(a_1,b_1)))$,
   $a_1 \prec s \prec a_3,\ a_2 \prec e \prec c$.

2. Traffic lights:

   $\forall t(light(c_1,t) \neq 0)$, $\forall t(light(c_2,t) \neq 0)$,
   $\forall p,t(p \neq c_1 \wedge p \neq c_2 \wedge$
   $(on(p,(a_1,b_1)) \vee on(p,(a_2,b_2)) \vee on(p,(a_3,b_3)) \Rightarrow$
   $light(p,t)=0)$.

3. Robot's initial state:

   $at(R,s,t_s)=1$,
   $face(R,orient(s,a_3),t_s)=1$,
   $move\_velocity(R,orient(a_1,a_3),t_s)>0$.

We can derive

1. There is no triffic light between s and c_1:
   $\forall p,t(s \prec p \prec c_1 \Rightarrow turn\_velocity(R,p,t)=0)$.

   – given 2

2. There is a time t_{c_1} when R is at c_1:
   $\exists t_{c_1}(\forall t(t_s < t < t_{c_1} \Rightarrow$
   $move\_velocity(R,orient(a_1,b_1),t)>0)\wedge$
   $at(R,c_1,t_{c_1})=1\wedge$
   $(move\_velocity(R,orient(a_1,b_1),t_{c_1})=0 \Leftrightarrow$
   $((light(c_1,t_{c_1})=1\wedge turn\_velocity(R,c_1,t_{c_1})=0)\vee$
   $(light(c_1,t_{c_1})\neq 1\wedge turn\_velocity(R,c_1,t_{c_1})\neq 0))))$.

   – given 2, 3, derived 1 and Axiom 3.7

3. Time t_{c_1} is bounded as follows:
   $t_{c_1} - t_s \leq |s,c_1|$
   $/min(t_s,t_{c_1},move\_velocity(R,orient(a_1,b_1),t))$.

   – given 3, derived 2 and Theorem 3.2

4. At time t_{c_1}, R is still facing b_1:
   $face(R,orient(a_1,b_1),t_{c_1})=1$.

   – given 3, derived 2 and Axiom 3.5

5. Over the time interval (t_s,t_{c_1}), R travels subpath (s,c_1):
   $\int_{t_s}^{t_{c_1}} move\_velocity(R,orient(a_1,b_1),t)dt = |s,c_1|$.

   – given 3, derived 2 and Definition 3.12

6. Path (s,c_1) meets and precedes path (c_1,c_2):
   $meet((s,c_1),(c_1,c_2)) \wedge (s,c_1) \preceq_{[s,e]} (c_1,c_2)$.

   – given 1 and Definition 3.6

7. There is a time t''_{c_1} such that over time interval (t_{c_1},t''_{c_1}) R turns from facing $orient(a_1,b_1)$ to facing $orient(a_3,b_3)$:
   $\exists t''_{c_1}(\int_{t_{c_1}}^{t''_{c_1}} turn\_velocity(R,c_1,t)dt =$
   $sub(orient(a_3,b_3),orient(a_1,b_1))\wedge$
   $\forall d,t(t_{c_1} \leq t < t''_{c_1} \Rightarrow (at(R,c_1,t)=1\wedge$
   $move\_velocity(R,d,t)=0))\wedge$
   $face(R,orient(a_3,b_3),t''_{c_1})=1\wedge$
   $move\_velocity(R,orient(a_3,b_3),t''_{c_1})>0)$.

   – given 3, derived 5, 6 and Axiom 3.9

8. If at time t_{c_1} the traffic light at place c_1 is green:
   (a) $light(c_1,t_{c_1})=2$.

   – assumption

   (b) At time t_{c_1}, R is turning at place c_1:
   $at(R,c_1,t_{c_1})=1\wedge$
   $turn\_velocity(R,c_1,t_{c_1})\neq 0$.

   – derived 8(a), 7 and 2

   (c) Time t''_{c_1} is bounded as follows:
   $t''_{c_1} - t_{c_1} \leq sub(orient(a_3,b_3),orient(a_1,b_1))$
   $/min(t_{c_1},t''_{c_1},turn\_velocity(R,c_1,t))$.

   – derived 4, 8(b), 7 and Theorem 3.3

9. If at time t_{c_1} the traffic light at place c_1 is red:
   (a) $light(c_1,t_{c_1})=1$.

   – assumption

   (b) At time t_{c_1}, R is waiting at place c_1:
   $at(R,c_1,t_{c_1})=1\wedge$
   $turn\_velocity(R,c_1,t_{c_1})=0\wedge$
   $move\_velocity(R,orient(a_1,b_1),t_{c_1})=0$.

– derived 9(a), 7 and 2

(c) There is a time t'_{c_1} when the traffic light at place c_1 is green:

$$\exists t'_{c_1}(t_{c_1} \leq t'_{c_1} < t_{c_1} + T \wedge light(c_1, t'_{c_1}) = 2).$$

– derived 9(a) and Theorem 3.1

(d) Over the time interval (t_{c_1}, t'_{c_1}), R is waiting:

$$\forall t(t_{c_1} < t < t'_{c_1} \Rightarrow (move\_velocity(R, orient(a_1, b_1), t) = 0 \wedge turn\_velocity(R, c_1, t_{c_1}) = 0)) \wedge (move\_velocity(R, orient(a_1, b_1), t'_{c_1}) \neq 0 \vee turn\_velocity(R, c_1, t'_{c_1}) \neq 0).$$

– derived 9(a), 9(b), 4, 9(c) and Axiom 3.8

(e) At time t'_{c_1}, R is still facing b_1:

$$face(R, orient(a_1, b_1), t'_{c_1}) = 1.$$

– derived 4, 9(d) and Axiom 3.4

(f) At time t'_{c_1}, R is turning at place c_1:

$$at(R, c_1, t'_{c_1}) = 1 \wedge turn\_velocity(R, c_1, t'_{c_1}) \neq 0.$$

– derived 7 and 9(d)

(g) Time t''_{c_1} is bounded as follows:

$$t''_{c_1} - t'_{c_1} \leq sub(orient(a_3, b_3), orient(a_1, b_1)) / min(t'_{c_1}, t''_{c_1}, turn\_velocity(R, c_1, t)).$$

– derived 9(e), 9(f), 9(g), 7 and Theorem 3.3

10. At time t''_{c_1}, R is still at c_1:

$$at(R, c_1, t''_{c_1}) = 1.$$

– derived 7, 2 and Axiom 3.3

11. There is a bounded time t''_{c_2} such that R is moving at place c_2 towards e:

$$\exists t''_{c_2}(at(R, c_2, t''_{c_2}) = 1 \wedge face(R, orient(a_2, b_2), t''_{c_2}) = 1 \wedge move\_velocity(R, orient(a_2, b_2), t''_{c_2}) > 0).$$

– derived 10, 7, given 1, 2, and apply derived 1 - 9 again

12. There is a bounded time t_e when R reaches e:

$$\exists t_e(t_e - t''_{c_2} \leq |c_2, e| / min(t''_{c_2}, t_e, move\_velocity(R, orient(a_2, b_2), t)) \wedge at(R, e, t_e) = 1).$$

– derived 11, given 1, 2 and Axiom 3.7, Theorem 3.2

4 Conclusions

Space and time must be taken into account when commonsense knowledge such as how to get from one place to another is represented in a computer. However, previous work [4, 3, 1] in this area either ignores the temporal feature of motion in space or does not give a proper description of spatial structure for reasoning about motion in space. In this paper we presented a spatio-temporal logic and showed how to represent the common knowledge of a two-dimensional street network with active traffic lights, and how to formalize the commonsense reasoning problem of going from one place to another in this street network. The proposed logic is also suitable for representing and reasoning about motion in three-dimensional world.

Acknowledgements

The first author is supported by an Acadia Graduate Fellowship. The second author is supported by NSERC grant OGP0046773.

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