Granular Computing: basic issues and possible solutions

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Abstract: *Granular computing (GrC) may be regarded as a label of theories, methodologies, techniques, and tools that make use of granules, i.e., groups, classes, or clusters of a universe, in the process of problem solving. The main objective of this paper is to discuss basic issues of GrC, with emphasis on the construction of granules and computation with granules. After a brief review of existing studies, a set-theoretic model of GrC is proposed based on the notion of power algebras.*

1 Introduction

Basic ideas of crisp information granulation have appeared in related fields, such as interval analysis, quantization, rough set theory, Dempster-Shafer theory of belief functions, divide and conquer, cluster analysis, machine learning, databases, and many others [16]. The topic of fuzzy information granulation was first proposed and discussed by Zadeh [14] in 1979. There is a fast growing and renewed interest in this topic [13]. Granular computing is likely to play an important role in the evolution of fuzzy logic and its applications.

1.1 What is GrC?

The following quotations from Zadeh may help us in understanding the scope of, and reasons for, granular computing (GrC):

> "Granulation of an object A leads to a collections of granules of A, with a granule being a clump of points (objects) drawn together by indistinguishability, similarity, proximity or functionality." (Zadeh [16], 1997)

> "The theory of fuzzy information granulation (TFIG) is inspired by the ways in which humans granulate information and reason with it." (Zadeh [16], 1997)

> "TFIG builds on the existing machinery of fuzzy information granulation in fuzzy logic but takes it to a significantly higher level of generality, consolidates its foundations and suggests new directions." (Zadeh [16], 1997)

> "GrC is a superset of the theory of fuzzy information granulation, rough set theory and interval computations, and is a subset of granular mathematics." (Zadeh [17], 1997)

It is clear that an underlying idea of granular computing is the use of groups, classes, or clusters of elements called granules [14, 16]. Although extensive work has been done on granular computing, it still might be difficult to give a precise definition. In this paper, we will not attempt to provide such a definition. Instead, we consider granular computing to be a label of theories, methodologies, techniques, and tools that make use of granules in the process of problem solving. Based on this intuitive understanding, we will investigate some basic issues and their possible solutions.

1.2 Why do we study GrC?

There are many reasons for the study of granular computing. From a philosophical and theoretical point of view, many authors argued that information granulation is very essential to human problem solving, and hence has a very significant impact on the design and implementation of intelligent systems. Zadeh [16] identified three basic concepts that underlie human cognition, namely, granulation, organization, and causation. "Granulation involves decomposition of whole into parts, organization involves integration of parts into whole, and causation involves association of causes and effects." Yager and Filev [7] pointed out that "human beings have been developed a granular view of the world", and "...objects with which mankind perceives, measures, conceptualizes and reasons are granular". From a more practical point of view, the necessity of information granulation and simplicity derived from information granulation in problem solving are perhaps some of the main reasons. In many situations, when a problem involves incomplete, uncertain, or vague information, it may be difficult to differentiate distinct elements and one is forced to consider granules. A typical example is the theory of rough sets [5]. The lack of information may only allow us to define granules rather than individuals. In some situations, although detailed information may be available, it may be sufficient to use granules in order to have an efficient and practical solution. In fact, very precise solutions may not be required at all for many practical problems. It may also happen that the acquisition of precise information is too costly, and coarse-grained information reduces cost.

In summary, the rationales of granular computing suggests the basic guiding principle of fuzzy logic:

> "*Exploit the tolerance for imprecision, uncertainty and partial truth to achieve tractability, robustness, low solution cost and better rapport with reality.*"

This principle offers a more practical philosophy for real world problem solving. Instead of searching for the optimal solution, one may search for good approximate solutions. One only needs to examine the problem at a finer granulation level with more detailed information when there is a need or benefit for doing so. However, it should be pointed out that studies of granular computing are only complementary to vigorous investigations on precise and non-granular computational approaches. The latter may provide justifications and guidelines for the former.

1.3 What are the basic issues of GrC?

Basic issues of granular computing may be studied from two related aspects, the construction of granules and computation with granules. The former deals with the formation, representation, and interpretation of granules, while the latter deals with the the utilization of granules in problem solving.

The interpretation of granules focuses on the semantics side of granule constructions. It addresses the question of *why* two objects are put into the same granule. Typically, elements in a granule are drawn together by indistinguishability, similarity, proximity, or functionality [16]. Furthermore, information granulation depends on the available knowledge. In the construction of granules, it is necessary to study criteria for deciding if two elements should be put into the same granule, based on available information. In other words, one must provide necessary semantics interpretations for notions such as indistinguishability, similarity, and proximity. It is also necessary to study granulation structures derivable from various granulations of the universe [13]. The formation and representation of granules deal with algorithmic issues of granule construction. They address the problem of *how* to put two objects into the same granule. Algorithms need to be developed for constructing granules efficiently.

Computation with granules can be similarly studied from both the semantic and algorithmic perspectives. On the one hand, one needs to interpret and interpret various relationships between granules, such as closeness, dependency, and association, and to define and interpret operations on granules. On the other hand, one needs to design methodologies and tools for computing granules, such as approximation, reasoning, and inference.

Both semantics and algorithmic aspects of granular computing are important. However, many existing methods of granular computing do not pay enough attention to the semantics aspect. It is equally, if not more, important to investigate semantics issues involved in granular computing. The results may provide not only interpretations and justifications for a particular GrC model, but also guidelines that prevent possible misuses of the model.

2 Overview of Two GrC Models

A clearer picture of granular computing may be obtained by examining some particular models.

2.1 Zadeh's formulation

A general framework of granular computing was given in a recent paper by Zadeh [16] based on fuzzy set theory. Granules are constructed and defined based on the concept of generalized constraints. Relationships between granules are represented in terms of fuzzy graphs or fuzzy if-then rules. The associated computation method is known as computing with words (CW) [7, 15].

Let X be a variable taking values in a universe U. A generalized constraint on the values of X can be expressed as $X\ isr\ R$, where R is a constraining relation, isr is a variable copula and r is a discrete variable whose value defines the way in which R constrains X. Examples of constraints are equality, possibilistic, probabilistic, fuzzy, and veristic constraints. For example, an equality constraint, $r = e$, is given by $X\ ise\ a$, which means $X = a$. A possibilistic constraint, $r = blank$, is given by $X\ is\ R$, where R is a possibility distribution of X. With the introduction of generalized constraints, a granule is defined by a fuzzy set:

$$G = \{X \mid X\ isr\ R\}. \quad (1)$$

Depending on the types of constraints, various classes of granules can be obtained. From simple granules, one may obtain Cartesian granules by considering combinations of constraints [16].

One may label granules by natural language words. This establishes a basis for computing with words. As one of the core components of fuzzy logic, CW deals with fuzzy if-then rules of the form:

$$if\ X\ isr_1\ A\ then\ Y\ isr_2\ B, \quad (2)$$

where r_1 and r_2 may represent different types of constraints although the same type is commonly used. A set of fuzzy if-then rules can be interpreted in terms of a fuzzy graph. Inference can be carried out using fuzzy if-then rules, or fuzzy graphs [15, 16].

2.2 Pawlak's rough sets

With the granulation of universe, one considers elements within a granule as a whole rather than individually [14]. The loss of information through granulation implies that some subsets of the universe can only be approximately described. The theory of rough sets deals mainly with the approximation aspect of information granulation [5].

Let $E \subseteq U \times U$ denote an equivalence relation on the universe U. The pair $apr = (U, E)$ is called an approximation space. The equivalence relation E partitions the set U into disjoint subsets known as the quotient set U/E. Each equivalence class may be viewed as a granule consisting of indistinguishable elements, and it is also referred to as an equivalence granule. A particular semantic interpretation of equivalence relations is provided based on the notion of information tables. Two objects are equivalent if they have exactly the same value with respect to a set of attributes. Thus, an equivalence granule is characterized by an equality constraint [13].

An arbitrary set $X \subseteq U$ may not necessarily be a union of some equivalence classes. This implies that one may not be able to describe X precisely using the equivalence classes of E. In this case, one may characterize X by a pair of lower and upper approximations:

$$\underline{apr}(X) = \bigcup_{[x]_E \subseteq X} [x]_E, \quad \overline{apr}(X) = \bigcup_{[x]_E \cap X \neq \emptyset} [x]_E, \quad (3)$$

where $[x]_E = \{y \mid xEy\}$ is the equivalence class containing x. The lower approximation $\underline{apr}(X)$ is the union of all the equivalence granules which are subsets of X. The upper approximation $\overline{apr}(X)$ is the union of all the equivalence granules which have a non-empty intersection with X.

Based on approximations of sets, one may perform data analysis and mining tasks in information tables, such as attribute reduction, dependency analysis, and learning of decision rules [5].

3 A Set-theoretic Model of GrC

In this section, we present a set-theoretic formulation of granular computing. Each granule represents certain concept such that each element of the granule is an instance of the concept. Granules can be constructed through the use of information tables, as being done in rough set approach [5, 13]. We therefore concentrate mainly on operations on granules. The use of crisp sets (granules) is not as restrictive as it may appear. A fuzzy set (granule) can be equivalently expressed a family of crisp sets (granules) using its α-cuts. Operations on fuzzy granules can therefore be defined by operations on α-cuts.

3.1 Power algebras

Let $\circ$ be a binary operation on a universe U. One can define a binary operation $\circ^+$ on subsets of U as follows [1]:

$$X \circ^+ Y = \{x \circ y \mid x \in X, y \in Y\}, \quad (4)$$

for any $X, Y \subseteq U$. In general, one may lift any operation f on elements of U to an operation f^+ on subsets of U, called the power operation of f. Suppose $f : U^n \longrightarrow U$ $(n \geq 1)$ is an n-ary operation on U. The power operation $f^+ : (2^U)^n \longrightarrow 2^U$ is defined by [1]:

$$f^+(X_0, \ldots, X_{n-1}) = \{f(x_0, \ldots, x_{n-1}) \mid x_i \in X_i \text{ for } i = 0, \ldots, n-1\}, \quad (5)$$

for any $X_0, \ldots, X_{n-1} \subseteq U$. This provides a universal-algebraic construction approach. For any algebra $(U, f_1, \ldots, f_k)$ with base set U and operations $f_1, \ldots, f_k$, its power algebra is given by $(2^U, f_1^+, \ldots, f_k^+)$.

The power operation f^+ may carry some properties of f. For example, for a binary operation $f : U^2 \longrightarrow U$, if f is commutative and associative, f^+ is commutative and associative, respectively. If e is an identity for some operation f, the set $\{e\}$ is an identity for f^+. If an unary operation $f : U \longrightarrow U$ is an involution, i.e., $f(f(x)) = f(x)$, f^+ is also an involution. On the other hand, many properties of f are not carried over by f^+. For instance, if a binary operation f is idempotent, i.e., $f(x, x) = x$, f^+ may not be idempotent. If a binary operation g is distributive over f, g^+ may not be distributive over f^+.

3.2 Interval number algebra

An *interval number* $[\underline{a}, \overline{a}]$ with $\underline{a} \leq \overline{a}$ is the set of real numbers defined by:

$$[\underline{a}, \overline{a}] = \{x \mid \underline{a} \leq x \leq \overline{a}\}. \quad (6)$$

Degenerate intervals of the form $[a, a]$ are equivalent to real numbers.

One can perform arithmetic operations on interval numbers by lifting arithmetic operations on real numbers [4]. Let $A = [\underline{a}, \overline{a}]$ and $B = [\underline{b}, \overline{b}]$ be two interval numbers, we have:

$$\begin{aligned} A + B &= \{x + y \mid x \in A, y \in B\} \\ &= [\underline{a} + \underline{b}, \overline{a} + \overline{b}], \end{aligned}$$

$$
\begin{aligned}
A - B &= \{x - y \mid x \in A, y \in B\} \\
&= [\underline{a} - \overline{b}, \overline{a} - \underline{b}], \\
A \cdot B &= \{x \cdot y \mid x \in A, y \in B\} \\
&= [\min(\underline{a}\,\underline{b}, \underline{a}\,\overline{b}, \overline{a}\,\underline{b}, \overline{a}\,\overline{b}), \\
&\quad \max(\underline{a}\,\underline{b}, \underline{a}\,\overline{b}, \overline{a}\,\underline{b}, \overline{a}\,\overline{b})], \\
A/B &= \{x/y \mid x \in A, y \in B\} \\
&= [\underline{a}, \overline{a}] \cdot [1/\overline{b}, 1/\underline{b}], \quad 0 \notin [\underline{b}, \overline{b}]. \qquad (7)
\end{aligned}
$$

The results of interval number operations are again closed and bounded intervals. When $0 \in B$, A/B is undefined. One may lift any operations on real numbers, such as min and max, to power operations on intervals of real numbers [11].

Interval number algebra may serve as a basis for interval reasoning with numeric truth values, such as interval fuzzy reasoning [11], interval probabilistic reasoning [6, 9], and reasoning with granular probabilities [2]. It can be easily extended to study fuzzy arithmetic with fuzzy numbers [3].

3.3 Interval set algebra

Given two subsets $A_1, A_2 \in 2^U$ with $A_1 \subseteq A_2$, the subset of 2^U,

$$
\mathcal{A} = [A_1, A_2] = \{X \in 2^U \mid A_1 \subseteq X \subseteq A_2\}, \qquad (8)
$$

is called a closed *interval set* [8]. The set A_1 is called the lower bound of the interval set and A_2 the upper bound. Degenerate interval sets of the form $[A, A]$ are equivalent to ordinary sets.

By lifting standard set-theoretic operations, such as intersection $\cap$, union $\cup$ and difference $-$, we define interval set operations as follows: for two interval sets $\mathcal{A} = [A_1, A_2]$ and $\mathcal{B} = [B_1, B_2]$,

$$
\begin{aligned}
\mathcal{A} \sqcap \mathcal{B} &= \{X \cap Y \mid X \in \mathcal{A}, Y \in \mathcal{B}\}, \\
&= [A_1 \cap B_1, A_2 \cap B_2], \\
\mathcal{A} \sqcup \mathcal{B} &= \{X \cup Y \mid X \in \mathcal{A}, Y \in \mathcal{B}\}, \\
&= [A_1 \cup B_1, A_2 \cup B_2], \\
\mathcal{A} \setminus \mathcal{B} &= \{X - Y \mid X \in \mathcal{A}, Y \in \mathcal{B}\} \\
&= [A_1 - B_2, A_2 - B_1]. \qquad (9)
\end{aligned}
$$

The results of interval set operations are also interval sets. The interval set complement $\neg[A_1, A_2]$ of $[A_1, A_2]$ is defined by $[U, U] \setminus [A_1, A_2]$. This is equivalent to $[U - A_2, U - A_1] = [\sim A_2, \sim A_1]$.

The set algebra $(2^U, \cap, \cup, \sim)$ is a special Boolean algebra. By using the same argument, one can lift operations in a Boolean algebra or a lattice [11]. Such interval algebras may be used for reasoning with interval extension of classical logic [10], and interval incidence calculus [12].

4 Conclusion

Granular computing may have a great impact on the design and implementation of intelligent information systems, and on real world problem solving. The results from existing studies show the richness and flexibility of GrC. They also suggest that further research, especially on the semantics aspect of GrC, is needed. The set-theoretic model may provide a simple framework for the study of GrC.

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