Chen, Y.H. and Yao, Y.Y., Multiview intelligent data analysis based on granular computing
Proceedings of 2006 IEEE International Conference on Granular Computing, pp. 281-286, 2006.

Multiview Intelligent Data Analysis based on Granular Computing

Yaohua Chen, Yiyu Yao

***Abstract*— Multiview intelligent data analysis explores data from different perspectives to reveal various types of structures and knowledge embedded in the data. Granular computing provides a general methodology for problem solving and information processing. Its application to data analysis results in hierarchical knowledge structures. In this paper, the fundamental issues of granulations and granular structures for data analysis are discussed based on modal-style data operators. The results provide a basis for establishing a framework of multiview intelligent data analysis.**

I. Introduction

Techniques and models of data mining, machine learning, knowledge discovery, and statistics can be applied to intelligent data analysis [12]. Typically, each model and method presents a particular and single view of data or discovers a specific type of knowledge embedded in the data. By combining existing studies, it is possible to investigate a data set from multiple views. This leads to the introduction of multiview intelligent data analysis. It explores different types of knowledge, different features of data, and different interpretations of data. Multiple aspects of data understanding, multiple angles of data summarizations or data descriptions, and multiple types of discovered knowledge are crucial for satisfying a wide range of needs of a large diversity of users.

Granular computing has emerged as a multi-disciplinary study of problem solving and information processing [16], [17], [24], [27], [30]. It provides a general, systematic and natural way to analyze, understand, represent, and solve real world problems. Its methodologies can be applied to intelligent data analysis. Granulations and granular structures are two fundamental issues in granular computing. Different types of granulations and granular structures reflect multiple aspects of data [27]. Granulation involves the construction of granules. The structures of all granules illustrate the relationships or connections among the granules. Different granular structures describe different characteristics of data or knowledge embedded in data.

Many researchers studied modal-style data operators for data analysis [8], [10], [11], [21], [22], [25], [26]. In this paper, we use those operators to form granules and granular views. Multiview intelligent data analysis is modeled as the investigation of multiple types of granulations and granular structures.

Authors are with the Department of Computer Science, University of Regina, Regina, Saskatchewan, Canada S4S 0A2, Email: {chen115y, yyao}@cs.uregina.ca

II. Motivations

Multiple view strategies and approaches have been proposed and studied in many fields, including social sciences, software design, machine learning, and data analysis [1], [2], [14], [18], [32]

Jeffries and Ransford argued that the study of social stratification should not focus on isolated and fragmented views, but holistic and unified views [14]. They proposed a multiple hierarchy model to integrate ethnicity, sex, and age hierarchies for social stratification analysis.

Belkhouche and Lemus-Olalde studied the formal foundations of an abstract interpretation of multiple views at the software design stage [1], [2]. Each view is a design that describes specific features of a system and is represented by one or more design notations. The multiple view analysis framework is used to compare views and identify the discrepancies among different views.

Several researchers proposed frameworks of multistrategy learning to integrate a wide range of learning strategies [18]. They argued that the research on multistrategy systems is significantly relevant to study on human learning since human learning is clearly multistrategy, and multistrategy systems have a potential to be more versatile and more powerful to solve a much wider range of learning problems than monostrategy systems.

Zhong proposed an approach of multiple aspect analysis of human brain data for investigating human information processing [31], [32]. They argued that every method of data analysis has its own strength and weakness, and analyzing the data from multiple aspects for discovering new models of human information processing is necessary.

A multiple view approach would be more suitable for intelligent data analysis. An integrated and unified framework that allows a multiple view approach on the understanding, computation, and interpretation of data is needed. Furthermore, studies of different views, their connections and transformations are also important.

III. Overview of Granular Computing

Basic notions of granular computing are subsets, classes and clusters of universe [24], [30]. Granulation of the universe, relationships of granules and computing with granules are three fundamental issues of granular computing.

Granulation of a universe involves the grouping of individual elements into classes and decomposition and combination of granules. It provides a granulated view of the universe.

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Elements in a granule are grouped together by indistinguishability, similarity, proximity or functionality [30]. One needs a semantic interpretation of why two objects are put into the same granule and how two objects are related with each other. Algorithms for a granulation should define how to put two objects into the same granule. The granules in a granulated view can be either disjoint or overlapping and are regarded as a level of the subject matter of the study.

There are many granulated views of the same universe. Different views of the universe can be linked together. A structure of linked granules can be established. The linkage of granules can be defined based on the relationships between granules. The relationships can be interpreted as ordering, closeness, dependency or association between granules.

The study of granulations and granular structures is the base for computing with granules. Many computational operations can be performed on granules such as reasoning, inferencing and learning.

Granular computing provides a general methodology for problem solving. The study of intelligent data analysis can be based on granular computing. For a data set, one can perform multiple granulations and construct multiple granular structures with respect to different strategies. Each of them provides a particular view of data and represents a specific type of knowledge.

IV. Multiview Intelligent Data Analysis

In this section, we investigate different types of granulations and granular structures. Based on modal-style data operators [11], [26], various types of granules are constructed.

A. Formal Contexts and Data Operators

We assume that a data set is given in terms of a formal context [21] or a binary table [20]. It provides a convenient way to describe a finite set of objects by a finite set of attributes [20].

Let U and V be any two finite sets. Elements of U are called objects, and elements of V are called attributes or properties. The relationships between objects and attributes are described by a binary relation R between U and V, which is a subset of the Cartesian product $U \times V$. For a pair of elements $x \in U$ and $y \in V$, if $(x, y) \in R$, written as xRy, we say that x has the attribute y, or the attribute y is possessed by object x. The triplet (U, V, R) is called a formal context [10], [21] or a binary information table [4]. In general, a multi-valued formal context can be transformed into a binary formal context through scaling [10]. That is, every information table can be represented by a formal context [3]. Table I is an example of a formal context.

Objects and attributes in a formal context are described and determined by each other. We assume that there does not exist an object without any attribute or an attribute not being shared by any objects. In other words, in a formal context, an object must have at least one attribute, and an attribute must be associated with at least one object.

Based on the binary relation R, we associate a set of attributes with an object. An object $x \in U$ has the set of attributes:

TABLE I

A formal context

	a	b	c	d	e
1	×		×	×	×
2	×		×		
3		×			×
4		×			×
5	×				
6	×	×			×

$$xR = \{y \in V \mid xRy\} \subseteq V.$$

The set of attributes xR can be viewed as a description of the object x. In other words, object x is described or characterized by the set of attributes xR.

Similarly, an attribute y is possessed by the set of objects:

$$Ry = \{x \in U \mid xRy\} \subseteq U.$$

By extending these notations, we can establish relationships between sets of objects and sets of attributes. This leads to two types of modal-style data operators, one from 2^U to 2^V and the other from 2^V to 2^U [8], [10], [11], [21], [22], [25], [26]. Different operators lead to different types of rules summarizing the relationships between data. They produce different methods to construct granules.

Definition 1: **(Basic Set Assignments)** The basic set assignment operators are defined by: for a set of objects $X \subseteq U$ and a set of attributes $Y \subseteq V$,

$$X^b = \{y \in V \mid Ry = X\},$$
$$Y^b = \{x \in U \mid xR = Y\}.$$

For simplicity, the same symbol is used for both operators. The operators ^{b} associate a set of objects with a set of attributes, and a set of attributes with a set of objects.

Definition 2: **(Sufficiency Operators)** The sufficiency operators are defined by: for a set of objects $X \subseteq U$ and a set of attributes $Y \subseteq V$,

$$X^* = \{y \in V \mid X \subseteq Ry\} = \bigcap_{x \in X} xR, \quad (1)$$

$$Y^* = \{x \in U \mid Y \subseteq xR\} = \bigcap_{y \in Y} Ry. \quad (2)$$

By definition, $\{x\}^* = xR$ is the set of attributes possessed by the object x, and $\{y\}^* = Ry$ is the set of objects having attribute y. For a set of objects X, X^* is the maximal set of properties shared by all objects in X. Similarly, for a set of attributes Y, Y^* is the maximal set of objects that have all attributes in Y.

Definition 3: **(Dual Sufficiency Operators)** The dual sufficiency operators are defined by: for a set of objects $X \subseteq U$

and a set of attributes $Y \subseteq V$,

$$X^{\#} = \{y \in V \mid X \cup Ry \neq U\},$$
$$Y^{\#} = \{x \in U \mid Y \cup xR \neq V\}.$$

For a subset $X \subseteq U$, an attribute in $X^{\#}$ is not possessed by at least one object not in X. The operators * and $^{\#}$ are dual to each other.

Definition 4: **(Necessity Operators)** The necessity operators are defined by: for a set of objects $X \subseteq U$ and a set of attributes $Y \subseteq V$,

$$X^{\Box} = \{y \in V \mid Ry \subseteq X\},$$
$$Y^{\Box} = \{x \in U \mid xR \subseteq Y\}.$$

By definition, an object having an attribute in $X^{\Box}$ is necessarily in X. The operators are referred to as the necessity operators.

Definition 5: **(Possibility Operators)** The possibility operators are defined by: for a set of objects $X \subseteq U$ and a set of attributes $Y \subseteq V$,

$$\begin{aligned} X^{\Diamond} &= \{y \in V \mid Ry \cap X \neq \emptyset\} \\ &= \bigcup_{x \in X} xR, \\ Y^{\Diamond} &= \{x \in U \mid xR \cap Y \neq \emptyset\} \\ &= \bigcup_{y \in Y} Ry. \end{aligned}$$

An object having an attribute in $X^{\Diamond}$ is only possibly in X. The operators are referred to as the possibility operators. The operators $^{\Box}$ and $^{\Diamond}$ are dual operators.

B. Granulations

Many types of binary relations among the objects have been studied [19], [28]. The relations between objects can be defined based on the relations between their sets of attributes. We consider four types of relations: equivalence relation, partial order relation, similarity relation and negative similarity relation, which can be formally defined by modal-style data operators.

Equivalence Relation. Two objects may be viewed as being equivalent if they have the same description [20]. An equivalence relation can be defined by: for $x, x' \in U$,

$$x \equiv x' \iff xR = x'R,$$

which is reflective, symmetric and transitive.

Partial Order Relation. A partial order relation on objects can be defined by set inclusion: for $x, x' \in U$,

$$x \preceq x' \iff xR \subseteq x'R,$$

which is reflective and transitive.

Similarity Relation: If two objects x and x' have some overlapping attributes, they are regarded as being similar to each other. This type of relation is defined by: for $x, x' \in U$,

$$x \approx x' \iff x'R \cap xR \neq \emptyset,$$

which is reflective and symmetric.

Negative Similarity Relation: For two objects x and x', if the union of their attributes is not the whole set of attributes, we can consider them as being negatively similar. This type of relation is defined by: for $x, x' \in U$,

$$\begin{aligned} x \asymp x' &\iff xR^c \cap x'R^c \neq \emptyset, \\ &\iff xR \cup x'R \neq V, \end{aligned}$$

which is symmetric, where R^c is the complement of the relation R.

A binary relation $\Re$ over a universe U is a subset of the Cartesian product $U \times U$. Based on a binary relation between objects, a 1-neighborhood system can be constructed [28]. For two object $x, x' \in U$, if $x \Re x'$, we say that x is a predecessor of x', and x' is a successor of x. The set of predecessors of x is called predecessor neighborhood and denoted as $\Re x = \{x' \in U \mid x'Rx\}$, and the set of successors of x is called successor neighborhood and denoted as $x\Re = \{x' \in U \mid xRx'\}$.

An object may have different types of predecessor and successor neighborhoods based on different types of binary relations. If a neighborhood for an object is considered as a granule, different binary relations induce different types of granules.

With equivalence relation $\equiv$, an object has the same predecessor and successor neighborhood. For an object $x \in U$, the equivalence class containing x is given by:

$$\begin{aligned} \equiv x &= \{x' \in U \mid x' \equiv x\}, \\ &= \{x' \in U \mid x \equiv x'\}, \\ &= x\equiv, \\ &= [x]. \end{aligned}$$

An equivalence class is viewed as a granule. This type of granule can be re-expressed by using modal-style data operators.

Definition 6: In a formal context (U, V, R), for an object $x \in U$, its equivalence class can be re-expressed by:

$$\begin{aligned} [x] &= \{x' \in U \mid x' \equiv x\}, \\ &= \{x' \in U \mid xR = x'R\}, \\ &= \{x' \in U \mid x'R = \{x\}^*\}, \\ &= \{x\}^{*b}. \end{aligned}$$

A partial order relation is not symmetric. In this case, we have different predecessor and successor neighborhoods for an object. This leads to two different types of granules.

Definition 7: In a formal context (U, V, R), for an object $x \in U$, the granule defined by the $\preceq$ successor neighborhood

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is given by:

$$\begin{aligned} x\preceq &= \{x' \in U \mid x \preceq x'\}, \\ &= \{x' \in U \mid xR \subseteq x'R\}, \\ &= \{x' \in U \mid \{x\}^* \subseteq x'R\}, \\ &= \{x\}^{**}. \end{aligned}$$

The set $\{x\}^{**}$ is the *maximal* set of objects that have *all* attributes in $\{x\}^*$. In other words, any objects in $\{x\}^{**}$ has at least all attributes of x. The set of objects in $\{x\}^{**}$ is in fact the extension of an object concept in formal concept analysis [10], [21].

Definition 8: In a formal context (U, V, R), for an object $x \in U$, the granule defined by the $\preceq$ predecessor neighborhood is given by:

$$\begin{aligned} \preceq x &= \{x' \in U \mid x' \preceq x\}, \\ &= \{x' \in U \mid x'R \subseteq xR\}, \\ &= \{x' \in U \mid x'R \subseteq \{x\}^*\}, \\ &= \{x\}^{*\Box}. \end{aligned}$$

The set $\{x\}^{*\Box}$ is the *maximal* set of objects whose attributes are subsets of $\{x\}^*$. In other words, any object in $\{x\}^{*\Box}$ has at most all attributes of x.

Based on the similarity relation $\approx$, for an object, its predecessor and successor neighborhoods are the same because of the symmetric property of $\approx$. Given a similarity relation, we can define a granule by using the predecessor or successor neighborhood of an object.

Definition 9: In a formal context (U, V, R), for an object $x \in U$, the granule defined by the $\approx$ predecessor or successor neighborhood is given by:

$$\begin{aligned} \approx x &= x\approx, \\ &= \{x' \in U \mid x \approx x'\}, \\ &= \{x' \in U \mid x' \approx x\}, \\ &= \{x' \in U \mid x'R \cap xR \neq \emptyset\}, \\ &= \{x\}^{*\Diamond}. \end{aligned}$$

The objects in $\{x\}^{*\Diamond}$ must share some attributes of x.

For the negative similarity relation $\asymp$, the predecessor and successor neighborhoods for an object are the same. Therefore, a granule can be defined by using the predecessor or successor neighborhood.

Definition 10: In a formal context (U, V, R), for an object $x \in U$, the granule defined by the $\asymp$ predecessor or successor is:

$$\begin{aligned} \asymp x &= x\asymp, \\ &= \{x' \in U \mid x \asymp x'\}, \\ &= \{x' \in U \mid x' \asymp x\}, \\ &= \{x' \in U \mid xR \cup x'R \neq V\}, \\ &= \{x\}^{*\#}. \end{aligned}$$

For an object x' in $\{x\}^{*\#}$, there must exist at least one attribute that is possessed by both x and x'.

These types of granules induce different types of granulated views of the universe. They can be considered as different types of strategies to divide the universe or classify the objects.

Based on the connections among various types of modal-style data operators [26], the connections among different types of granules can also be established. In fact, the equivalence classes can be used to re-express other types of granules.

$$\begin{aligned} \{x\}^{**} &= \bigcup\{[x'] \mid xR \subseteq x'R\}, \\ \{x\}^{*\#} &= \bigcup\{[x'] \mid xR \cup x'R \neq V\}, \\ \{x\}^{*\Box} &= \bigcup\{[x'] \mid xR \supseteq x'R\}, \\ \{x\}^{*\Diamond} &= \bigcup\{[x'] \mid xR \cap x'R \neq \emptyset\}. \end{aligned}$$

One can define operations on the granules to generate larger granules or smaller granules. Two basic operations on granules are combination and decomposition. The combination operation is to combine smaller granules into larger granules. The decomposition operation is to divide larger granules into smaller granules.

Larger granules can be expressed by smaller granules. For example, the following various types of granules can be expressed by equivalence classes.

$$\begin{aligned} X^{**} &= \bigcup\{[x] \mid X^* \subseteq xR\}, \\ X^{\#\#} &= \bigcup\{[x] \mid X^\# \cup xR \neq V\}, \\ X^{\Box\Diamond} &= \bigcup\{[x] \mid X^\Box \cap xR \neq \emptyset\}, \\ X^{\Diamond\Box} &= \bigcup\{[x] \mid xR \subseteq X^\Diamond\}. \end{aligned}$$

In fact they are the subjects of studying of formal concept analysis [25], [26]. All granules in a particular level provide a particular granulated view of data. The granules in different levels are related to each other. The granules and the relationships between them provide a whole picture of knowledge of data in a formal context.

The relationships between granules can be expressed in the form of rules. Quantitative measures can be associated with rules to indicate their strength. These rules and associated measures define many types of knowledge. Yao and Zhong analyzed and modeled various types of rules for data mining and knowledge discovery [29].

C. Granular Structures

Hierarchy and lattice of granules are two typical structures. Rough set analysis, hierarchical class analysis and formal concept analysis are three approaches of data analysis in which different hierarchical granular structures are proposed.

Rough Set Analysis: The objects in $[x]$ are considered to be indistinguishable from x. One is therefore forced to consider $[x]$ as a whole. In other words, under an equivalence relation, equivalence classes are the smallest non-empty observable, measurable, or definable disjoint subsets of U.

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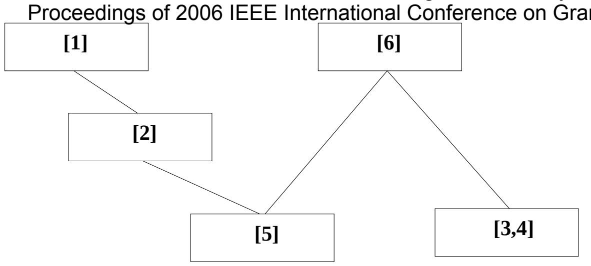

Fig. 1. Hierarchy of object classes for the context of Table I

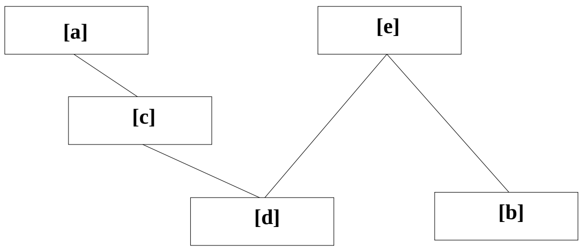

Fig. 2. Hierarchy of attribute classes for the context of Table I

The family of pair-wise disjoint equivalence classes is called the partition of a universe and denoted by $U/\equiv$. We can construct a larger definable set by taking a union of some equivalence classes. A definable set is viewed as a granule. The family of all definable sets contains the empty set $\emptyset$ and is closed under set complement, intersection and union. It is an σ-algebra $\sigma(U/\equiv) \subseteq 2^U$ with basis $U/\equiv$, where 2^U is the power set of U.

The intersection and union of definable sets in $\sigma(U/\equiv)$ are still definable sets. The system $\sigma(U/\equiv)$ under union ($\cup$) and intersection ($\cap$) is a complete lattice [13].

Hierarchical Class Analysis: Hierarchical class analysis is an approach on data analysis by studying hierarchical structures of data [4], [5], [6], [7], [9]. Granules (equivalence classes) in different levels can be linked by using the partial order relation.

The partial order relation among the granules is defined based on the partial order relation among the objects.

Definition 11: For two granules $[x]$ and $[x']$, the partial order relation between them is defined by:

$$[x] \preceq [x'] \Longleftrightarrow x \preceq x'.$$

Based on the partial order relation $\preceq$ over the family of granules $U/\equiv$, a hierarchical structure can be constructed. An example of granular hierarchy is illustrated in Figure 1. It is derived from Table I. Each rectangle represents a granule.

Moreover, the equivalence relation over the universe of attributes can produce the equivalence classes of attributes. A hierarchical structure of attribute granules can be built based on the similar partial order relation among the equivalence classes of attributes. The hierarchy of attribute granules derived from Table I is illustrated in Figure 2.

Formal Concept Analysis: Formal concept analysis is an approach to deal with a visual presentation and analysis of data [10], [21]. It follows the traditional notion of concept. Namely, a concept is defined jointly by an extension and an intension. The extension is a set of objects that are instances of a concept. The intension is a set of attributes that are possessed by instances of a concept. The extension and intension of a concept must uniquely determine each other. This leads to the notion of formal concept [10], [21].

Definition 12: A pair (X, Y), $X \subseteq U, Y \subseteq V$, is called a formal concept of the context (U, V, R), if $X^* = Y$ and $Y^* = X$.

In fact, a formal concept can be expressed by (X^{**}, X^*), $X \subseteq U$. The partial order relation between concepts can be defined based on either the extensions or the intensions.

Definition 13: For two formal concepts (X_1, Y_1) and (X_2, Y_2), (X_1, Y_1) is a sub-concept of (X_2, Y_2), written $(X_1, Y_1) \preceq (X_2, Y_2)$, and (X_2, Y_2) is a super-concept of (X_1, Y_1), if and only if $X_1 \subseteq X_2$, or equivalently, if and only if $Y_2 \subseteq Y_1$.

The super-concepts are more general than the sub-concepts because the super-concepts have less attributes and more objects (i.e. instances) than the sub-concepts. Conversely, the sub-concepts are more specific than the super-concepts.

The meet and join operations on formal concepts can be defined based on the meet and join operations in lattice theory [21].

Theorem 1: The meet and join among the formal concepts are given by:

$$\bigwedge_{t \in T} (X_t, Y_t) = (\bigcap_{t \in T} X_t, (\bigcup_{t \in T} Y_t)^{**}),$$
$$\bigvee_{t \in T} (X_t, Y_t) = ((\bigcup_{t \in T} X_t)^{**}, \bigcap_{t \in T} Y_t),$$

where T is an index set and for every $t \in T$, (X_t, Y_t) is a formal concept.

The extensions of formal concepts are considered as granules. Namely, X^{**}, $X \subseteq U$, is a granule. All formal concepts can be expressed by granules. That is, the family of all formal concepts, denoted as $L(U, V, R)$, is defined by:

$$L(U, V, R) = \{(X^{**}, X^*) \mid X \subseteq U\}.$$

The family of all formal concepts $L(U, V, R)$ under the meet ($\wedge$) and join ($\vee$) operations is a complete lattice. The concept lattice derived from Table I is illustrated in Figure 3.

A granule of objects can be expressed as a join of some small granules or a union of some equivalence classes of objects [10], [15]. For a formal concept (X, Y),

$$\begin{aligned} X &= \bigvee_{x \in X} \{x\}^{**}, \\ &= (\bigcup_{x \in X} \{x\}^{**})^{**}, \\ &= \bigcup_{X^* \subseteq xR} [x]. \end{aligned}$$

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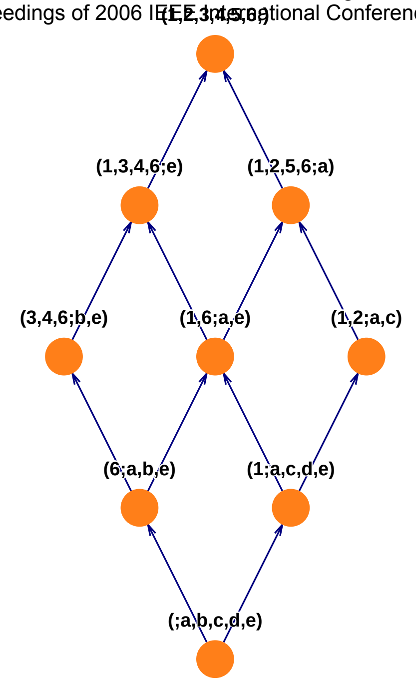

Fig. 3. Concept lattice for the context of Table 1, produced by "Formal Concept Calculator" (developed by Sören Auer, http://www.advis.de/soeren/fca/).

The meet and join operations in the lattice implement combination and decomposition of granules. In other words, meets of granules generates smaller granules, and joins of granules produces larger granules.

V. Conclusion

We investigate multiview intelligent data analysis based on granular computing. Different types of granulation and granular structure represent different aspects of data and provide different types of knowledge embedded in data. The results of this work produce a basis for research on multiview intelligent data analysis. The integration of these multiple views may provide more useful data analysis tools.

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