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Perspectives of Granular Computing

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***Abstract*— Granular computing emerges as a new multi-disciplinary study and has received much attention in recent years. A conceptual framework is presented by extracting shared commonalities from many fields. The framework stresses multiple views and multiple levels of understanding in each view. It is argued that granular computing is more about a philosophical way of thinking and a practical methodology of problem solving. By effectively using levels of granularity, granular computing provides a systematic, natural way to analyze, understand, represent, and solve real world problems. With granular computing, one aims at structured thinking at the philosophical level, and structured problem solving at the practical level.**

I. Introduction

The term "Granular Computing (GrC)" was first introduced in 1997 by T.Y. Lin to label a new field of multi-disciplinary study [11], [51]. Since then, we have witnessed a rapid development of and a fast growing interest in the topic [3], [8], [13], [15], [16], [17], [23], [24], [25], [26], [30], [31], [32], [35], [36], [47], [55]. Many methods and models of granular computing have been proposed and studied. The results enhance our understanding of granular computing.

Existing studies of granular computing typically concentrate on concrete models and computational methods in particular contexts. They unfortunately only reflect specific aspects of granular computing. In fact, there does not exist a formal, precise, commonly agreed, and uncontroversial definition of what is granular computing, nor there is a unified model. Consequently, the potential applicability and usefulness of granular computing are not well perceived and appreciated.

Since concrete models and methods of granular computing have been extensively studied by many authors, we focus on a high, conceptual level investigation in an attempt to address some of the fundamental issues. The main objective is to discuss some important perspectives of granular computing, based on our initial research on establishing a holistic, whole, and integrated view of granular computing [37], [38], [39], [40], [41], [42], [43], [44], [45], [46].

The discussion is divided into two parts. In the first part, we study perspectives of granular computing by trying to answer, at least partially, the following questions:

What is granular computing?
Why granular computing?
What is new and different?
What are the basic issues?

An examination of these questions enables us to derive, in the second part, a general framework of granular computing, based on hierarchy theory [1], [2], [19], [20], [27], [28], [29].

II. Granular Computing: What and Why

The principles and concepts of granular computing may be understood by answering a few fundamental questions.

A. *What is granular computing?*

This is perhaps the first question that comes into mind and is also one of most frequently asked questions. Many authors used the term granular computing without giving a precise definition, as it may be a difficult, if not impossible, task. Instead, one relies on an understanding of the term based on commonsense and intuition. Any specific definition may inevitably overlook some important aspects of granular computing, which may be counter-productive at the early development stage of the field.

Several fields contribute significantly to the study of granular computing. Many researchers in granular computing community formulate the problem based on theories and models of computational intelligence [3], [12], [13], [14], [15], [23], [25], [26], [30], [31], [34], [39], [47], [52]. In 1979, Zadeh first introduced the notion of information granulation and suggested that fuzzy set theory may find potential applications in this respect [49]. In 1982, Pawlak proposed the theory of rough sets [21], [22], which in fact provides a concrete example of granular computing. To some extent, rough set theory makes more people realize the importance of the notion of granulation. In 1997, Zadeh revisited information granulation [50], which led to a renewed interest. In the same year, Lin suggested the term granular computing to label the new and growing research filed [11], [51]. Moreover, Lin proposed to use neighborhood systems for the formulation of granular computing [12], [13], [14], [15], [37], which is an extension of the partition-based rough set theory.

Yao interpreted granular computing in a wide context based on the principles and ideas from other fields of computer science [43], [44]. The ideas of granular computing have been investigated in artificial intelligence through the notions of granularity and abstraction. Hobbs proposed a theory of granularity [7], which is similar to the theory of rough sets in terms of formulation. The theory indeed captures some of the essential features of granular computing. That is, we perceive and represent the world under various grain sizes, and abstract only those things that serve our present interests. The ability to conceptualize the world at different granularities and to switch among these granularities is fundamental to our intelligence and flexibility. This enables us to map the complexities of real world into computationally tractable simpler theories.

The principles of the theory of granularity have been applied in many studies. Giunchigalia and Walsh proposed a theory of abstraction [6]. Like the conceptualization in level of granularity, abstraction is a process for us to consider what is relevant and to ignore irrelevant details. Knoblock proposed a theory of hierarchical planning [10], in which plans of different granularities are considered. Zhang and Zhang developed a quotient space theory of problem solving based on hierarchical description and representation of a problem [53], [54]. The quotient space theory motivates us to view granular computing as a way of structured problem solving.

The connections between granular computing and hierarchy theory can also be established [44]. The hierarchy theory focuses on the understanding and representation of complex systems using multiple level structures [1], [2], [19], [20], [27], [28], [29], [33]. Hierarchical structure can be observed in many natural, artificial, and abstract systems. It reflects the orderness, control, and stability of such systems. One can conceptualize a complex system by discriminating entities, relations, processes and levels as the basic ingredients of a hierarchical structure. A hierarchy links the parts or components into a whole, and hence provides a multi-level and multi-resolution description of a system. In spite of some criticisms, hierarchical analysis is one of the successful methods used in the investigation and understanding of complex systems. The results from hierarchy theory can be used to develop a general framework of granular computing, which is to be explored later in this paper.

We view granular computing as a multi-disciplinary study with the objective to investigate and model the family of granule-oriented problem solving methods and information processing paradigms [38], [43]. It is a study of a general theory of problem solving based on different levels of granularity and detail [38], [43], [53], [54].

Our view is based on an underlying assumption that the basic principles and methodologies are common in most types of problem solving, independent of disciplines and problem domains. Granular computing, therefore, focuses on everyday and commonly used concepts and notions, such as granule, granulated view, granularity, and hierarchy. The notions of granular computing may be interpreted in terms of abstraction, generalization, clustering, levels of abstraction, levels of detail, and so on in various domains.

B. Why granular computing?

There are many reasons for the study of granular computing. The previous discussion provides some motivations. They stem mainly from the use of levels of granularity. The following list summarizes and reiterates some of the points:

1) **Truthful representation of the real world**. Many natural, social, and artificial systems are organized into levels [1], [29]. Granular computing provides true and natural representations of such systems. Through the multiple level representation, one can obtain a full understanding of a system.
2) **Consistent with human thinking and problem solving**. Human problem solving is based crucially on levels of granularity and change between granularities [7], [50]. Granular computing therefore extracts the common elements from human problem solving. The implementation of the principles of granular computing would lead to more effective information processing systems [53].
3) **Simplification of problems**. A multiple level representation shows the orderness, the control, and the organization of a complex system or a complex problem. Different levels focus on different granularities characterized by different grain sizes. By omitting unnecessary, irrelevant details and focusing on the right level of abstraction, we are able to simplify a complex system, or a complex problem.
4) **Economic and low cost solutions**. By considering the same problem at different levels of granularity, we ignore some details. This in turn may lead

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to approximate and inaccurate solutions [50]. A benefit is that such solutions can normally be obtained economically at a fraction of the cost.

In summary, the benefits of granular computing is evident from its basic guiding principle, which is given by Zadeh concisely as to "exploit the tolerance for imprecision, uncertainty and partial truth to achieve tractability, robustness, low solution cost and better rapport with reality" [50].

C. *What is new?*

It has been clearly recognized that the basic ideas of granular computing have been explored in many fields, such as artificial intelligence, interval analysis, quantization, rough set theory, Dempster-Shafer theory of belief functions, divide and conquer, cluster analysis, machine learning, programming, databases, and many others [43], [50]. Questions arise naturally: What is new in granular computing? What makes granular computing different from other existing studies?

The answer to the this question in fact lies on the existence of many theories. Each of them provides a set of concrete methods and tools for solving a particular type of problems and in a particular context. However, they are scattered in many fields with relatively little interaction. Granular computing, in our view, therefore attempts to extract the commonalities from those fields to establish a set of generally applicable principles, to synthesize their results into an integrated whole, and to connect fragmentary studies in a unified framework. It is this high level view, as well as the associated structured way of thinking and systematic way of problem solving, that is much needed, although concrete models and methods of granular computing are still going to be of great interest.

Granular computing can be studied by applying its principles and ideas. It can be investigated at different levels or from perspectives by focusing on its philosophical foundations, basic components, fundamental issues, and general principles. The philosophical level concerns structured thinking, and the application level deals with principles of structured problem solving. While structured thinking provides guidelines and leads naturally to structured problem solving, structured problem solving implements the philosophy of structured thinking.

The philosophy of thinking in terms of levels of granularity, and its implementation in more concrete models, would result in disciplined procedures that help to avoid errors and to save time for solving a wide range of complex problems.

III. A General Framework of Granular Computing

A detailed discussion of basic issues of granular computing is given recently elsewhere [43], [44]. Some of the basic notions of granular computing are granules, levels, and hierarchies. This section presents a general framework by drawing results from hierarchy theory. It briefly summarizes a review on hierarchy theory by Yao *et al*. [48], but within the context of granular computing. It should be recognized that hierarchical analysis and granular computing share many things in common.

A. *Overview*

A hierarchy is simply viewed as a family of stratified levels, without further constraints. A hierarchy represents different levels of granularity in granular computing. There are two important issues of the simple definition. The basic ingredients of a hierarchy are levels, and furthermore the levels are linked together by a partial order. A level is populated by, or consists of, granules (or entities) whose properties characterize the level. Levels may be considered as parts, and the partial order describes the relations between, or dependency of, parts. Under the partial order, parts are arranged inside a whole described by a hierarchy.

A level itself can be a hierarchy, which in turn consists of many levels. Conversely, one may combine levels into a hierarchy as one level in the original hierarchy. The combination and decomposition processes add more expressive power of the notion of hierarchy.

The general principles of hierarchical analysis and granular computing, understanding of the whole in terms of its parts and understanding of the system based on its inherent internal structures, are almost universally applicable. In practice, one also needs to consider the particular features of a system or project. When describing a specific system, one may impose on additional system dependent constraints and interpretations.

B. *Granules and levels*

Granules, levels, and relationships between them are the basic ingredients of granular computing. Granules populate at a particular level. They are the subjects of investigation at that level. Different levels focus on different, though related, types of granules. The properties of granules collectively characterize a level of description and understanding. Levels are connected together through a partial order. Granules in different levels are related to each other.

The physical meanings of granules, levels, and their relationships become clearer when considering more concrete systems. A number of fundamental issues must be addressed. Some questions are listed below [2], [5], [19], [20], [28]:

What generates the levels?
How many levels are needed?
Why are the levels discrete?
What separates the levels?
What ties the levels together?
How do levels interact with each other?

These questions can not be answered satisfactorily without reference to a particular system and domain specific knowledge. Some of the questions may not be answered at all with the current state of knowledge. However, it is still possible to present some general remarks.

Pattee [20] suggested that hierarchical systems may be characterized by the requirement of levels of description and the requirement of the levels of structure. The requirement of the levels of structure captures inherent natural of a complex system, and the requirement of the levels of description captures our understanding of the complex system. This offers two extreme views for the interpretation of a hierarchy [20]. At one extreme, it is viewed that the structural levels of matter determined by the entirely objective laws of nature. We adopt the corresponding levels of description for easiness and convenience of analysis. The other extreme focuses on the human subjective multi-level understanding of the reality. A hierarchy is formulated and built by levels of description of our choice, which is based on our understanding through laws of the nature and the results of our observations. The relation between the structural and descriptive levels is a crucial issue of hierarchical analysis.

The partial order implies that there are at least two ways for the construction of a hierarchy, namely the top-down and the bottom-up approaches. One can explain wholes by decomposing them into smaller and smaller parts, based on analytic thinking. Such an approach is typically used by physicists to explain the physical world around us. Alternatively, one can construct wholes from smaller parts, based on synthetic thinking.

Interpretations of the partial order of a hierarchy are of fundamental importance. As a matter of fact, the levels of structure and levels of description are more general interpretations. Different, and more concrete, interpretations of the partial order can also be considered. A few additional interpretations are given below:

1) **Levels of abstraction**. Different levels of abstraction may represent different granulated views of our understanding of a real world problem [7], [43], [53].
2) **Levels of reduction**. This interpretation is related to reductionism in science, which believes that the fully understanding and explanation of a phenomenon can be gained by reducing it to its constituents or other phenomena that are more basic or fundamental [4], [29]. Reduction allows us to derive the laws governing the entities at each higher level from the laws governing the entities at its next-lower level. The so-called bridge principles link entities at a lower level with entities, or structures of entities, at its higher level. The bridge principles capture the interactions of different levels.
3) **Levels of control**. Many social hierarchies and systems are governed by levels of control. It is reasonable to assume that a higher level governs its lower levels, and lower levels are subordinate to their higher levels.
4) **Levels of detail**. In the implementation of information systems, levels of detail play an important role. A computer software system is normally designed and implemented top-down by adding more details in a step-wise manner [5], [18].

Other interpretation of levels are available, such as the levels of explanation, the levels of difficulty, the levels of observation, levels of organization, and so on. Allen [2] presented more concrete examples for the ordering of levels: a higher level may be the context of, offer constraints to, behave more slowly at a lower frequency than, be populated by granules with greater integrity and higher bond strength than, and contain and be made of, lower levels.

It is desirable that the levels are, to some degree, relative independent. The connections and interaction between two adjacent levels are expressed in terms of bridge principles. By the transitivity of the partial order, the connections and interactions between any two levels can be achieved through the composition of bridge principles.

C. Multiple hierarchies and multiple levels

In granular computing, we stress holistic, unified views, in contrast to isolated, fragmented views. To achieve this, we need to consider multiple hierarchies and multiple levels in each hierarchy. An example of such a multiple hierarchy approach was given by Jeffries and Ransford in the study of social stratification [9].

The multiple interpretations of the order relation of levels implies that one is able to derive multiple hierarchies for the same system. Each hierarchy is defined based on a particular interpretation of the order relation.

With each hierarchy represents a particular aspect, one obtain multiple aspects that characterize the same system.

With respect to a particular hierarchy, levels represent localized views, and they are tied together by bridge principles to form a global view. Based on the bridge principles, one may climb up and down the hierarchy to study a system at various resolution.

Multiple hierarchies can be tied together by the conjunction of multiple hierarchies and the break-down of a hierarchy into many hierarchies. A level itself can be a hierarchy with many levels and one may combine levels into a hierarchy as a single level in the original hierarchy. Such combination and decomposition processes ties together multiple hierarchies in another way.

The power and effectiveness of hierarchy theory, as well as its flexibility and generality, stem from this combination of multiple hierarchies and multiple levels. Granular computing explores such potentials.

IV. Conclusion

We elaborate on several issues of the granular computing by attempting to answer its fundamental questions. Specific and concrete theories, methodologies, and tools of granular computing are discussed by many authors. For this reason, we concentrate on a high-level conceptual investigation. Based on our investigation, a general framework is presented by drawing results from hierarchy theory.

The investigation of this paper is exploratory in natural. The aim is to present broad perspectives of the problem at a high level without bearing down by unnecessary details. It perhaps raises more questions, rather than offering answers.

Granular computing, in our view, attempts to extract the commonalities from existing fields to establish a set of generally applicable principles, to synthesize their results into an integrated whole, and to connect fragmentary studies in a unified framework. Granular computing at philosophical level concerns structured thinking, and at the application level concerns structured problem solving. While structured thinking provides guidelines and leads naturally to structured problem solving, structured problem solving implements the philosophy of structured thinking.

The presented personal views and ideas may be immature, and perhaps controversial. They are meant to stimulate more researchers to look further into granular computing.

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