# Notes 03-6: Regression

*Linear regression* techniques attempt to model data using a straight line.

Given a set of data points (*x1*, *y1*), (*x2*, *y2*), …, (*xn*, *yn*), where *yi* is some response corresponding to *xi*, linear regression is a method for determining the function that best fits the observed data points.

The first step in fitting a straight line to the data points is to construct a scatter plot.

If the points appear to approximate a straight line, linear regression may be an appropriate analysis technique.

DIAGRAM = Classification.C.1.b1

If they don’t, some other technique is required.

DIAGRAM = Classification.C.1.b2

The method of least squares assumes the best-fit curve is one that has the minimal sum of the deviations squared from a given set of data points.

The general regression equation can be written as

where *α* and *β* are called the regression coefficients.

The regression coefficients can be estimated from the following two equations:

Where *x̅* is the mean of the *x* values in the sample, *y̅* is the mean of the *y* values, *β* represents the slope of the line through the points, and *α* represents the y-intercept.

Example – Linear regression

Consider the table shown below, where *Salary* is shown for various values of *Years of Service*. The objective is to use the data in this table to predict *Salary* based upon *Years of Service*. *Salary* is called the *explanatory* variable and *Years of Service* is called the *response* variable.

|  |  |
| --- | --- |
| Salary | Years of Service |
| 30 |  3 |
| 57 |  8 |
| 64 |  9 |
| 72 | 13 |
| 36 |  3 |
| 43 |  6 |
| 59 | 11 |
| 90 | 21 |
| 20 |  1 |
| 83 | 16 |

A scatter plot corresponding to the values in the table is shown below.

DIAGRAM = Classification.C.1.d

Based upon the values in the table, *x̅* = 9.1, *y̅* = 55.4, *β* = 3.54, and *α* = 23.19. Therefore *ŷ* = 23.19 + 3.54*x*.

*Salary* can now predicted for any value of *Years of Service*. However, keep in mind that it is just a prediction. For example, the actual versus predicted *Salary* for *Years of Service* from the original table is shown below.

|  |  |  |
| --- | --- | --- |
| Salary | Years of Service |   |
| 30 |  3 | 33.81 |
| 57 |  8 | 51.51 |
| 64 |  9 | 55.05 |
| 72 | 13 | 69.21 |
| 36 |  3 | 33.81 |
| 43 |  6 | 44.43 |
| 59 | 11 | 62.13 |
| 90 | 21 | 97.53 |
| 20 |  1 | 26.73 |
| 83 | 16 | 79.83 |

When interpreting the regression coefficients:

The estimated slope *β* = 3.54 implies that each additional year of service results in an increase in salary of $3,450.

The regression line should not be used to predict the response *ŷ* when *x* lies outside the range of the initial values.

Example

DIAGRAM = Classification.C.1.e

## Coefficient of Determination

The *coefficient of determination* represents the proportion of the total variability that is explained by the model.

The coefficient of determination is represented by

where the numerator is the measure of the total variability of the fitted values and the denominator is the measure of the total variability of the original values.

A value close to 1 implies that most of the variability is explained by the model.

A value close to 0 implies that the model is not appropriate.

## Naïve Bayes

The *Naïve Bayes* classifier is a well-known and highly effective classifier based upon *Bayes’ Rule*, a technique used to estimate the likelihood of class membership of an unseen instance given the set of labeled instances.

The *prior* (or *unconditional*) probability, *P*(*a*), associated with a proposition *a* (i.e., an assertion that *a* is true) is the degree of belief accorded to it in the absence of any other information.

Example – Prior probability

*P*(*rain* = true) = 0.25 or *P*(*rain*) = 0.25

The *posterior* (or *conditional*) probability, *P*(*a* | *b*), associated with a proposition *a* is the degree of belief accorded to it given that all we know is *b*.

Example – Posterior probability

*P*(*rain* | *thunder*) = 0.8

A prior probability, such as *P*(*rain*), can be thought of as a special case of the posterior probability *P*(*rain* | ), where the probability is conditioned on no evidence.

Posterior probabilities can be defined in terms of prior probabilities. Specifically,

for *P*(*b*) > 0, which can also be written as

In a nutshell: For *a* and *b* to be true, we need *b* to be true, and we need *a* to be true given *b*.

Since conjunction is commutative , so

can be written equivalently as

Then, since , we have Bayes’ Rule

which can be written as

A Naïve Bayes classifier applies to classification tasks where each instance *x* is described by a conjunction of attribute values (i.e., a tuple ) and where the target class can take on any value from some finite set *C* (i.e., the set of possible class values).

A set of labeled instances is provided from which the prior and posterior probabilities can be derived.

## Predicting with Naïve Bayes

When a new instance is presented, the classifier is asked to predict the class label.

The Bayesian approach considers a set of candidate hypotheses (i.e., the various possible class labels) and determines the hypothesis (i.e., the class label) that is most probable given the labeled instances (known as the *maximum posteriori hypothesis* (*MAP*)).

Given a new instance with attribute values , the most probable class label is given by

Using Bayes’ Rule, the above expression can be written as

or

That is, the denominator can be dropped because it is a constant term independent of *Cj*.

Since a Naïve Bayes classifier assumes the effect of an attribute value on a given class is independent of the values of the other attributes (called the *class conditional independence assumption*), given *CMAP*, the probability of observing the conjunction is just the product of the probabilities of the individual attributes. That is,

Substituting for in the equation for *CMAP* yields

where *CNB* denotes the assigned class label output by the Naïve Bayes classifier.

## Naïve Bayes Classifier for Categorical Attributes

Algorithm: Naïve Bayes Learner

Input: *D* = a set of labeled instances of the form ,

 where each *ai* corresponds to a value from the domain of

 attributes *A1*, *A2*, ..., *An*, respectively, and *an* is the

 assigned class label

Output: classProbability = an array of prior probabilities

 attributeProbability = an array of posterior probabilities

 v = an array of the number of unique values in the domain of each attribute

Method:

1. totalCount = 0
2. m = the number of unique classes in the domain of *An*
3. n = the number of attributes in the instances of *D*
4. for j = 1 to m
5. classCount [j] = 0
6. for i = 1 to n - 1
7. v [i] = the number of unique values in the domain of *Ai*
8. for k = 1 to v [i]
9. attributeCount [j, i, k] = 0
10. for each instance of *D*
11. totalCount ++
12. j = an integer corresponding to the class of the current instance
13. classCount [j] ++
14. for i = 1 to n - 1
15. k = an integer corresponding to the value of the current attribute
16. attributeCount [j, i, k] ++
17. for j = 1 to m
18. classProbability [j] = classCount [j] / totalCount
19. for i = 1 to n
20. for k = 1 to v [i]
21. attributeProbability [j, i, k] = attributeCount [j, i, k] / classCount [j]

Algorithm: NaiveBayesClassifier

Input: classProbability = an array of prior probabilities

 attributeProbability = an array of posterior probabilities

 m = the number of unique classes in the domain of *An*

 n = the number of attributes in the instances of *D*

 v = an array of the number of unique values in the domain of each attribute

 = an unlabeled instance

Output: *CNB* = the class label

Method:

1. *CNB* = 0
2. for j = 1 to m
3. *CTemp* = classProbability [j]
4. for i = 1 to n - 1
5. for k = 1 to v [i]
6. if *ai* == the attribute value corresponding to v [i]
7. *CTemp* = *CTemp* \* attributeProbability [j, i, k]
8. break
9. if *CTemp* > *CNB*
10. *CNB* = *CTemp*

Example – Predicting a class label using a Naïve Bayes classifier

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Tuple | Age | Income | Student | Credit Rating | Buys Computer |
| *t1* | <=30 | *high* | *no* | *fair* | *no* |
| *t2* | <=30 | *high* | *no* | *excellent* | *no* |
| *t3* | 31..40 | *high* | *no* | *fair* | *yes* |
| *t4* | >40 | *medium* | *no* | *fair* | *yes* |
| *t5* | >40 | *low* | *yes* | *fair* | *yes* |
| *t6* | >40 | *low* | *yes* | *excellent* | *no* |
| *t7* | 31..40 | *low* | *yes* | *excellent* | *yes* |
| *t8* | <=30 | *m* | *no* | *fair* | *no* |
| *t9* | <=30 | *low* | *yes* | *fair* | *yes* |
| *t10* | >40 | *medium* | *yes* | *fair* | *yes* |
| *t11* | <=30 | *medium* | *yes* | *excellent* | *yes* |
| *t12* | 31..40 | *medium* | *no* | *excellent* | *yes* |
| *t13* | 31..40 | *high* | *yes* | *fair* | *yes* |
| *t14* | >40 | *medium* | *no* | *excellent* | *no* |

The class label attribute is Buys Computer and it has two unique values: *yes* and *no*. The unlabeled instance to be classified is

〈Age = “<=30”, Income = *medium*, Student = *yes*, Credit Rating = *fair*〉.

Let *a1* = “<=30”, *a2* = *medium*, *a3* = *yes*, and *a4* = *fair*. So, the problem is to determine for all *j*. Now,

*P*(*C1*) = *P*(Buys Computer = *yes*) = 9/14

and

*P*(*C2*) = *P*(Buys Computer = *no*) = 5/14.

To determine *CNB*, we only need to concern ourselves with the conditional probabilities associated with the attribute values on the unlabeled instance. So,

 = *P*(*C1*) *P*(*a1*|*C1*) *P*(*a2*|*C1*) *P*(*a3*|*C1*) *P*(*a4*|*C1*)

 = (9/14)(2/9)(4/9)(6/9)(6/9)

 = (0.643)(0.222)(0.444)(0.667)(0.667)

 = 0.028

and

 = *P*(*C2*) *P*(*a1*|*C2*) *P*(*a2*|*C2*) *P*(*a3*|*C2*) *P*(*a4*|*C2*)

 = (5/14)(3/5)(2/5)(1/5)(2/5)

 = (0.357)(0.6)(0.4)(0.2)(0.4)

 = 0.007

We need to maximize . Therefore, *CNB* = *C1* = (Buys Computer = *yes*).