$$\begin{bmatrix} a_{11} & a_{21} & a_{31} & a_{41} \\ a_{12} & a_{22} & a_{32} & a_{42} \\ a_{13} & a_{23} & a_{33} & a_{43} \\ a_{14} & a_{24} & a_{34} & a_{44} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{21} & b_{31} & b_{41} \\ b_{12} & b_{22} & b_{32} & b_{42} \\ b_{13} & b_{23} & b_{33} & b_{43} \\ b_{14} & b_{24} & b_{34} & b_{44} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{21} & c_{31} & c_{41} \\ c_{12} & c_{22} & c_{32} & c_{42} \\ c_{13} & c_{23} & c_{33} & c_{43} \\ c_{14} & c_{24} & c_{34} & c_{44} \end{bmatrix}$$

$$c_{ij} = a_{1i}b_{j1} + a_{2i}b_{j2} + a_{3i}b_{j3} + a_{4i}b_{j4} = \sum_{k=1}^{4} a_{ki}b_{jk}$$

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) & 0 \\ 0 & \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_z(90) \times R_y(-90) \times R_x(90)$:

$$\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotate about the point (7,2,5):

$$\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & -7 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 1 & -5 \\ 0 & -1 & 0 & 2 \\ 1 & 0 & 0 & -7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & -1 & 0 & 4 \\ 1 & 0 & 0 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

How can we check our work?
Midterm exam question and solution:
Rotate about the point (5,-7,2)

$$\begin{bmatrix}1 & 0 & 0 & 5\\0 & 1 & 0 & -7\\0 & 0 & 1 & 2\\0 & 0 & 0 & 1\end{bmatrix} \times \begin{bmatrix}0 & 0 & 1 & 0\\0 & -1 & 0 & 0\\1 & 0 & 0 & 0\\0 & 0 & 0 & 1\end{bmatrix} \times \begin{bmatrix}1 & 0 & 0 & -5\\0 & 1 & 0 & 7\\0 & 0 & 1 & -2\\0 & 0 & 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}1 & 0 & 0 & 5\\0 & 1 & 0 & -7\\0 & 0 & 1 & 2\\0 & 0 & 0 & 1\end{bmatrix} \times \begin{bmatrix}0 & 0 & 1 & -2\\0 & -1 & 0 & -7\\1 & 0 & 0 & -5\\0 & 0 & 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}0 & 0 & 1 & (-2+5)\\0 & -1 & 0 & (-7+-7)\\1 & 0 & 1 & (-5+2)\\0 & 0 & 0 & 1\end{bmatrix}$$

Transformation Matrix:

$$\begin{bmatrix}0 & 0 & 1 & 3\\0 & -1 & 0 & -14\\1 & 0 & 1 & -3\\0 & 0 & 0 & 1\end{bmatrix}$$

Verify that the matrix leaves the centre of rotation unchanged:

$$\begin{bmatrix}5\\-7\\2\\1\end{bmatrix} = \begin{bmatrix}2+3\\7-14\\5-3\\1\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 3\\0 & -1 & 0 & -14\\1 & 0 & 1 & -3\\0 & 0 & 0 & 1\end{bmatrix} \times \begin{bmatrix}5\\-7\\2\\1\end{bmatrix}$$