Pet Activity Classification with Game-theoretic Shadowed Sets

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ABSTRACT

Monitoring and classifying pet activities is crucial for promoting pet health and early detection of behavioral or medical concerns. Existing computational intelligence techniques, including machine learning methods like neural networks and reasoning methods like fuzzy logic, have shown promise but struggle to effectively handle uncertainty. variability in pet behaviors, and sensor noise in real-world environments. In order to handle these issues, we propose a pet activity classification model based on the game-theoretic shadowed sets framework, which uses game-theoretic principles to dynamically determine the thresholds for the shadowed sets. The approach processes motion data collected from sensors embedded in wearable pet collars and adaptively classifies pet behavior into calm and active states. Experimental evaluation using real-world pet activity datasets demonstrated that the proposed model achieved an average classification accuracy of 91.17% across 10-fold cross-validation. The significant contribution of this research lies in introducing a flexible, interpretable, and uncertainty-aware classification model capable of dynamically adjusting to varying pet behaviors, enabling more accurate and reliable real-time pet activity monitoring systems.

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Chapter 1

INTRODUCTION

Understanding and classifying pet activity is vital for ensuring the health, safety, and overall quality of life of companion animals. Many pet owners today lead busy lives and are often unable to monitor their pets throughout the day. This lack of supervision can result in unnoticed health problems, undetected anxiety, or insufficient physical activity, which may affect the pet's long-term well-being. For instance, a dog that remains unusually calm for extended periods may be experiencing lethargy, pain, or underlying health conditions such as arthritis or heart disease [1]. Conversely, continuous hyperactivity in dogs has been associated with separation anxiety and stress-related behaviors, as studies have shown significant links between hyperactivity-impulsivity traits and separation-related symptoms in domestic dogs [2,3]. If such behavioral states go undetected, they may lead to delayed medical treatment, behavioral disorders, or even life-threatening conditions in some cases [38].

A smart classification system capable of distinguishing between active and calm states can support timely intervention, provide reassurance to owners, and serve as a decision-support mechanism in veterinary diagnostics. Furthermore, it can enable intelligent automation in pet-care environments, such as triggering alerts, adjusting indoor climate controls, or activating entertainment systems when pets exhibit signs of distress. Without such systems, pet owners are left with manual monitoring methods, which are often inaccurate and inconsistent [5]. Missed behavior patterns may lead to late diagnoses and reduced quality of care. To support this goal, the system outputs a detailed daily or weekly summary of the dog's activity, showing how much time is spent in calm and active states, helping users detect unusual behavior trends early.

Researchers have investigated various techniques to address the problem of pet activity recognition. Early efforts involved manually defined thresholds and expert-driven rules. Accelerometers have been widely used in animal science for various applications, including the estimation of energy expenditure, monitoring behaviors in free-ranging animals, and supporting veterinary diagnostics and health evaluations [4]. In contrast, machine learning models such as convolutional neural networks have been developed to process sensor data from wearable devices. For instance, Kasnesis et al. proposed a wearable device utilizing deep convolutional neural networks to process audio and motion data from search-and-rescue dogs, obtained an F1-score more than 99% in real-time activity recognition scenarios [6]. Similarly, Wang et al. introduced a hierarchical fuzzy model to detect separation anxiety in dogs, successfully interpreting complex behaviors such as pacing and vocalization [7].

Despite recent advancements, pet activity classification methods still face significant challenges due to uncertainty in behaviors, sensor inaccuracies, and data ambiguity in dynamic environments [8]. Traditional methods often struggle to manage incomplete or ambiguous data, leading to potential misclassification. Game-theoretic shadowed sets, which combine game theory with shadowed sets theory [41, 53], provide a structured framework to address these limitations [9]. This research proposes a game-theoretic shadowed sets-based approach to classify pet activities into active and calm states. The method analyzes sensor data from wearable collars, applies Gaussian membership functions to represent behavior patterns, and dynamically adjusts thresholds using game-theoretic strategies. The game-theoretic shadowed sets model incorporates a three-way decision-making structure to handle uncertainty explicitly, enabling flexible and interpretable classifications. Experiments using real-world datasets demonstrate the model's ability to classify ambiguous pet behaviors effectively, supporting more accurate and context-aware pet monitoring solutions.

The remainder of this report is organized as follows. Chapter 2 presents a review of related work in the field of pet activity classification. Chapter 3 introduces the background knowledge, including shadowed sets, game theory, and the game-theoretic shadowed sets framework. Chapter 4 describes the classification model based on the game-theoretic shadowed sets framework and explains how it is applied to pet activity recognition. Chapter 5 details the implementation process, including data handling, model development, and system design. Chapter 6 reports the experimental results and provides an in-depth analysis. Finally, Chapter 7 concludes the report and outlines directions for future research.

Chapter 2

LITERATURE REVIEW

Classifying pet activity is challenging due to the unpredictable nature of pet behavior and the noise often found in sensor data. To overcome these issues, researchers have developed various methods that help make better decisions when the data is uncertain. These include strategies like three-way decision-making, rough set models, and shadowed sets based on game theory. This review looks at how these different methods have been used to improve classification performance. It organizes the research into three categories: methods that involve human input, fully automated machine learning models, and other alternative techniques, highlighting their value and practical outcomes in pet activity monitoring.

2.1 Semi-Automated Pet Activity Classification Using Labeled Data

Early research in pet activity classification significantly depended on human intervention, particularly for data annotation and validation processes. For example, Ladha et al. (2013) developed a wearable collar device with an accelerometer sensor to monitor the daily activities of dogs in real-life conditions. The study focused on classifying 17 different dog behaviors related to their health and well-being. They collected data from 18 dogs and applied a statistical classification model to recognize activities, such as walking, sitting, and running. Their method achieved an accuracy of around 70% [30]. However, the approach required human involvement for labeling the data, making it a human-intervened machine learning method suitable for small-scale monitoring but limited for fully automated systems.

Kiyohara et al. (2015) developed a dog activity recognition system using a 3D accelerometer attached to the dog's collar. The main aim of their study was to recognize seven different activities performed by dogs. They collected data from 24 dogs during a semi-controlled environment and extracted various movement features from the sensor data. Their method achieved a maximum accuracy of 75.1% in classifying dog activities such as walking, sitting, standing, and lying [31]. However, this approach needed manual labeling and feature extraction, so it required human effort to train the model.

Human-intervened machine learning approaches, despite their effectiveness, demonstrated limitations concerning scalability and real-time deployment. The dependency on human efforts for continuous annotation posed challenges for widespread adoption and limited the applicability in dynamic, real-time pet monitoring scenarios. Thus, these methods necessitated advancements towards automation to mitigate human-related constraints.

2.2 Fully Autonomous Pet Activity Classification Systems

Recent developments have significantly shifted towards fully automated machine learning techniques, aiming to reduce or eliminate the dependency on human intervention. These approaches primarily involve sophisticated algorithms like deep learning models, capable of autonomously extracting relevant features directly from raw sensor data. Hussain et al. applied convolutional neural networks (CNN) to classify pet activities, after applying the class-weight balancing technique, achieved an high training accuracy of 99.70% and a validation accuracy of 96.85% [33].

Aich et al. developed a fully automated system to monitor dog activities and emotional behavior using wearable sensors that collected accelerometer and gyroscope data [34]. The system was tested on data collected from 10 dogs of various breeds, sizes, and ages, with real-time sensor signals recorded in parallel with video footage captured at frames per second to serve as ground truth for evaluation. Multiple machine learning algorithms were tested, including artificial neural networks (ANN), support vector machines, random forest, KNN, and naïve Bayes. Their best results achieved 96.58% accuracy for activity detection and 92.87% for emotional behavior recognition. This system proved that a combination of sensor data and robust machine learning techniques can enable real-time, automated pet monitoring across diverse dog types and settings.

Similarly, Wang et al. developed an automatic system to detect abnormal behaviors in dogs when they are alone at home [32]. The main focus of their study was to monitor stress-related activities that could lead to separation anxiety in pets. They used a wearable device with a built-in accelerometer sensor that collected movement data from the dogs at a high frequency of 50Hz. This movement data was analyzed using an LSTM (Long Short-Term Memory) model, which is a type of deep learning technique suitable for working with time-series data. Their method showed excellent performance, achieving 96% accuracy in detecting abnormal behaviors, proving that deep learning models like LSTM are highly effective for real-time dog behavior monitoring.

2.3 Pet Activity Classification with Game-theoretic Shadowed Set

Game-theoretic shadowed sets integrate shadowed set theory with game-theoretic strategies to enhance decision-making under uncertainty [53]. Unlike conventional classification models, game-theoretic shadowed sets are designed to handle ambiguous and overlapping data more flexibly by dividing information into three regions: acceptance, rejection, and uncertainty. The model dynamically adjusts decision thresholds through game-based optimization that balances classification accuracy and uncertainty.

Although game-theoretic shadowed sets provide a strong mathematical foundation for managing uncertainty, their direct application in pet activity classification has not yet been fully explored in the existing literature. Most current studies on pet activity recognition continue to rely on traditional machine learning or deep learning methods for classification tasks. Recent advancements, however, have extended the game-theoretic shadowed sets framework into various domains, demonstrating its adaptability and robustness. For instance, Jiang et al. proposed a shadowed set-based multi-granular three-way clustering ensemble (S-M3WCE), which integrates possibilistic C-means clustering and shadowed set theory to manage noise and uncertainty in unsupervised learning [26]. Their approach improves clustering quality through multi-granularity and three-way decision-making.

Similarly, Zhang and Yao introduced a novel game-theoretic formulation that enables threshold initialization from arbitrary values rather than the fixed (1, 0) pair, enhancing the flexibility and convergence behavior of the traditional game-theoretic shadowed sets model [54]. This approach provides dynamic threshold refinement through strategic learning, which is particularly beneficial in imbalanced or uncertain environments. Furthermore, Zhang applied the game-theoretic shadowed sets framework to TF-IDF-based term selection in text classification, where threshold selection was modeled as a strategic game [55]. Their results showed improved classification accuracy and computational efficiency compared to traditional heuristic methods.

These developments illustrate the versatility of game-theoretic shadowed sets in addressing classification challenges across different domains. However, their potential in pet activity classification—where data is often noisy, uncertain, and transitional—remains underexplored. Applying game-theoretic shadowed sets in this context presents a valuable research opportunity to develop more flexible, interpretable, and reliable behavior recognition models using wearable sensor data.

2.4 Evaluation of Pet Activity Monitoring

The study and classification of pet behavior is of interest to researchers and pet owners, especially in companion pets like dogs. Traditionally, knowledge of a dog's behavioral states, like rest and activity, was based on direct observation and subjective interpretation. Initial behavioral research required manual recording and analysis of activity patterns, which was time-consuming and prone to errors by the person performing them. Advances in wearable technology, such as with accelerometers and GPS trackers, have helped researchers and owners of pets obtain real-time information on the different movements an pet performs, which helps in conducting more extensive and continuous observations. Such technologies have laid a basis for automated activity classification, enabling quick analysis of large data sets to derive insights into behavior patterns.

Accurately classifying a dog's activity level, such as distinguishing between calm and active states, is very important in numerous applications. Such information would be useful to pet owners for ensuring appropriate exercise and rest for their dogs. For veterinarians, activity state classification provides support to the monitoring of health conditions, recovery processes, and management related to weight or stress issues. Accurate classification is important for researchers studying pet behavior, since it can yield important information related to a dog's physiological and psychological welfare. The movement from observational methods to data-driven monitoring highlights a growing need for methods that not only accurately classify behaviors but also elegantly handle the messy subtleties of pet movement, in which states often blur or shift in gradual ways.

Developments in machine learning have given significant impetus to activity monitoring, introducing marked improvements in accuracy and operational efficiency. Different algorithms of machine learning, like decision trees, support vector machines, and neural networks, are used for the classification of different conditions in pet behavior. These algorithms are normally trained on a labeled dataset to learn the patterns of the activities of pets so that they can be applied to real-time monitoring systems. However, such ambiguous or transitional behaviors may pose problems for traditional machine learning approaches, especially in cases where it is not clear whether a dog is wholly resting or active. These intermediate states are problematic to classify into a binary model, and the misclassifications reduce the reliability of the model.

The review highlights the importance of advancing dog activity monitoring systems for better pet care. Despite progress, challenges persist, motivating further innovation. Our study aims to contribute this progress by proposing a novel monitoring approach addressing these challenges and providing user-friendly tools for pet owners.

Chapter 3

THEORETICAL KNOWLEDGE

This chapter presents the theoretical foundations that support this study. It begins by introducing game theory, which models strategic interactions and decision-making among rational agents. It then explores shadowed sets, a framework for handling uncertainty by dividing data into clear, vague, and uncertain regions. Finally, it integrates these two concepts into game-theoretic shadowed sets (GTSS), which combine the strategic reasoning of game theory with the uncertainty modeling of shadowed sets. The chapter aims to build a solid conceptual understanding of these models, setting the stage for their application in pet activity classification.

3.1 Game Theory

Game theory, a mathematical framework for analyzing strategic interactions among rational decision-makers, was first developed by mathematicians John von Neumann and Oskar Morgenstern in the 1940s [36]. It gives us insight into situations where the outcome of a decision depends not only on one's own actions but also on the behavior of other people. Game theory has since found applications in economics, political science, biology, and computer science, influencing the development of strategies in competitive, cooperative, and conflictual situations [39]. Game theory has evolved, over time, to become a key tool in understanding strategic behavior in complex situations, where participants or decision-makers interact in a major way that can affect collective outcomes.

Some of the basic concepts of game theory include strategic interaction, payoff functions, and equilibrium analysis. In strategic interaction, each player's decision affects the final outcome; hence, there is usually some form of interdependency among players. The payoff is the result or reward a player gets depending on other players' decisions and their own. Nash equilibrium, a concept introduced by John Nash in 1950, refers to a state where no player can improve their payoff by changing their strategy while considering the strategies of others [37]. This equilibrium concept is central to game theory, providing a foundation for analyzing competitive and cooperative decision-making scenarios, as players settle into stable strategies where no single participant has an incentive to deviate unilaterally.

Game theory distinguishes between cooperative and non-cooperative games. In cooperative games, players can form coalitions and collaborate to achieve mutual benefits, whereas non-cooperative games assume that players act independently, each seeking to maximize their own payoff. Both types of games are used to model a variety of real-world scenarios, from market competition to international diplomacy [46]. In zero-sum games, one player's gain is exactly another's loss, while in non-zero-sum games, players may benefit mutually from cooperation, allowing for more complex and varied outcomes.

The application of game theory has expanded into machine learning and artificial intelligence, where it plays a vital role in decision-making and classification tasks. In particular, game-theoretic models have been used to improve machine learning algorithms, especially when faced with uncertainty or ambiguous decision boundaries. For example, game theory is used in multi-agent systems, where agents (software programs or robots) must make decisions in a shared environment while considering the actions of other agents [48]. These models are particularly useful in classification tasks, where decisions often need to account for not only the data at hand but also handle possible misclassifications carefully.

In the context of pet activity classification, such as differentiating between calm and active states in dogs, game theory can provide a structured approach to managing uncertainty and optimizing decision thresholds. Traditional classification methods often struggle with ambiguous data, particularly in situations where there is no clear distinction between classes. Game-theoretic approaches, including game-theoretic shadowed sets, integrate decision-making strategies to handle such uncertainty by defining flexible boundaries that adapt to the data [53]. These frameworks combine game theory's focus on strategic decision-making with fuzzy set theory, allowing for three-way classification that accounts for transitional or uncertain states. By using game theory, classifiers can adjust to the data and make more accurate and reliable decisions, even when it is hard to see clear boundaries between classes.

Overall, game theory has proven to be a powerful tool in optimizing decision-making, particularly in the presence of uncertainty. Its principles allow for strategic decision-making in competitive environments and have been successfully integrated into machine learning frameworks, enhancing the ability to classify complex behaviors. In applications like animal activity classification, game-theoretic models offer a robust solution for managing uncertainty and ambiguity, providing more effective and adaptable systems for real-world classification challenges.

3.2 Shadowed Sets

Shadowed sets reveal interesting conceptual and algorithmic relationships existing between rough sets and fuzzy sets [41]. Traditional binary classification methods assign data points to one of two predefined classes using a certain threshold. In countless practical situations, however, the data points do not clearly belong to either class—applications with noisy, ambiguous, or transitional data. Shadowed sets respond to this problem by creating a tertiary, intermediate area in which the ambiguity of classification can be manipulated systematically. The concept is particularly helpful in complex systems in which precise decision limits are problematic to define.

In a shadowed set, the decision space is divided into three different regions:

1. Acceptance Zone: Elements that are confidently classified as belonging to a particular class.

2. Non-Acceptance Zone: Elements that are confidently classified as not belonging to that class.

3. Shadowed Region: An uncertain zone where data points do not clearly belong to either class and require further analysis or additional rules for classification.

The shadowed region plays a critical role in managing the uncertainty between these two clear-cut zones, making shadowed sets an ideal framework for handling classification tasks involving fuzzy or ambiguous data. Shadowed sets, originally developed by Pedrycz [41], extend fuzzy set theory by introducing three regions for data classification: *Acceptance Zone*, *Non-Acceptance Zone*, and a *Shadowed Region*. Each data point d_i is assigned a membership value $\mu(d_i)$, computed through a fuzzy membership function. The membership of an object to a shadowed set is defined as follows [53]:

$$S_{(\alpha,\beta)}(\mu_A(x)) = \begin{cases} 1, & \mu_A(x) \geq \alpha \\ 0, & \mu_A(x) \leq \beta \\ [0,1], & \beta < \mu_A(x) < \alpha \end{cases} \tag{3.1}$$

Shadowed sets represent a valuable approach within computer science for managing uncertainty and imprecision in data. They have been applied across a range of domains. In the context of granular computing, shadowed sets facilitate the formation of information granules, supporting robust decision-making even when data is incomplete or imprecise [42, 49]. In clustering analysis, which involves grouping similar data points, shadowed sets enhance boundary detection and enable more flexible and interpretable partitioning of clusters [28, 43, 50, 51]. Additionally, they have been employed in image processing, where they contribute to improved interpretation of visual data [16,35], and in data analysis, where they aid in handling vague or fuzzy information [24,52]. Collectively, these applications highlight the effectiveness of shadowed sets in computational tasks that involve ambiguity and the need for soft decision boundaries.

Shadowed Sets Algorithm:

1. Initialization: Initialize a data set with each data point having a membership value.
2. Set Thresholds: Determine thresholds for:
   - Acceptance Zone: Membership $> \alpha$,
   - Zone of Non-Acceptance: Membership $< \beta$,
   - Shadowed Region: Membership between α and β,
3. Classify Data Points:
   - If membership $> \alpha$, classify as accepted.
   - If membership $< \beta$, classify as rejected.
   - If membership is between α and β, classify as shadowed (uncertain).
4. Handle Shadowed Region: Apply other decision-making techniques (such as game the-

ory or fuzzy rules) to resolve ambiguity. Assign each item to one of the three marked areas: accepted, not accepted or shadowed.

In the next chapter, this theoretical foundation of shadowed sets will be extended through integration with game-theoretic principles. Specifically, we will show how the concept of a shadowed region is operationalized in the classification of pet activity using adaptive thresholding and strategic error minimization. This allows the model to make flexible, uncertainty-aware decisions that go beyond rigid binary classification.

3.3 Game-theoretic Shadowed Sets

Game-theoretic shadowed sets are an approach to decision-making, combining methodologies of game theory and shadowed sets to deal with classification problems under uncertainty [53]. The methodology is especially useful in situations where data can't be obviously partitioned into binary groups and the divisions between the groups are fuzzy or imprecise. The adoption of principles of strategic decision-making originating from game theory combined with flexible classification frameworks belonging to shadowed sets gives rise to game-theoretic shadowed sets a powerful tool for dynamic decision-making in complex environments.

Components of game-theoretic shadowed sets:

At the core of shadowed sets, there lies the idea of breaking down a decision space into three different regions: acceptance, non-acceptance, and a shadowed region (also called an uncertain region). These three regions reflect the level of confidence a decision-maker has in classifying a data point to a class. Data points falling within the acceptance region are classified with high confidence as belonging to a particular class. In the non-acceptance zone, the data points are unambiguously classified as not belonging to that particular class. The shaded region is what lies between the acceptance and non-acceptance zones: this is a region of classification ambiguity, where a decision cannot be made with high confidence.

Incorporating game theory into this framework enhances the decision-making process by allowing the system to strategically select decision boundaries. Game theory provides a set of tools to model and optimize these boundaries based on minimizing misclassification costs, uncertainty, and other factors like the distance between the data points and classification thresholds. By using game-theoretic principles such as Nash equilibrium and minimax optimization, game-theoretic shadowed sets dynamically adjust the thresholds for classification to minimize error costs, handle ambiguity, and account for varying levels of uncertainty in the data [53].

Application in Classification Tasks

A significant advantage of game-theoretic shadowed sets is their ability to handle transitional states or uncertain data—situations in which a data point does not unambiguously belong to a particular class, which is very common in many real-life classification problems. For example, in classifying dog behaviors into calm and active classes, dogs can be in intermediate states that do not clearly belong to either class. In such cases, traditional classification methods based on binary labels tend to face problems in making accurate decisions. The game-theoretic shadowed sets approach tackles this issue by allowing the classifier to recognize and control uncertainty in the data by introducing the shadowed region, which helps regulate state transitions between active and calm.

By applying game-theoretic methods, this model succinctly decreases the cost of misclassification while optimizing decision boundaries to improve overall classification accuracy. In this way, the classifier becomes able to smoothly adapt itself to changes in the underlying data by changing thresholds based on the amount of uncertainty and the context in which each decision is made. This is especially advantageous in domains such as the classification of animal behavior, where data from real-time monitoring may be subject to noise or ambiguity. Accurate classification is essential for various tasks, including health assessment, behavioral research, and activity observation.

Advantages and Benefits

The game-theoretic shadowed sets framework offers several key advantages over traditional classification methods:

1. Handling Uncertainty: By incorporating a shadowed region, the system can handle uncertainty in data classification, especially when decisions are not clear-cut.

2. Adaptability: The game-theoretic approach allows the model to dynamically adjust decision thresholds based on the context and uncertainty of the data, improving the model's flexibility.

3. Cost Optimization: Game theory helps minimize the cost of misclassification by strategically setting thresholds that balance accuracy with cost, ensuring more efficient and reliable decision-making.

4. Robustness in Transitional States: The ability to handle ambiguous or transitional states, such as a dog moving from calm to active behavior, ensures more accurate and continuous monitoring.

Disadvantages

1. Computational Cost: Game-theoretic shadowed sets rely on the integration of game theory and fuzzy logic, which increases the overall model complexity. Due to this, GTSS may require significantly more computational time and resources for both training and inference, particularly when applied to large datasets.

2. Limited Tool Support: In contrast to commonly used machine learning algorithms such as Support Vector Machines or Neural Networks, game-theoretic shadowed sets lacks widespread implementation in standard machine learning libraries. As a result, developers often need to manually implement the methodology, which increases the development effort and may require advanced mathematical and programming knowl-

edge.

3. Difficult to Explain to Non-Experts: The theoretical basis of game-theoretic shadowed sets involves advanced concepts from both game theory and fuzzy set theory, making it challenging to interpret and communicate. This can present difficulties when explaining the model's behavior and outcomes to end-users, stakeholders, or domain experts who do not have a technical background.

Overall, the Game-Theoretic Shadowed Sets framework offers a major improvement in decision-making methods. It provides a practical and reliable approach to handling uncertainty and enhances classification accuracy, especially in real-time applications like pet behavior monitoring.

Chapter 4

Applying Game-theoretic Shadowed Sets to Pet Activity Classification

The game-theoretic shadowed sets framework unifies shadowed sets with game-theoretic principles for handling uncertainty in classification problems. Unlike binary classification, the game-theoretic shadowed sets framework has developed a flexible three-way decision model that treats the ambiguous data points with three different zones: *acceptance*, *rejection*, and a *shadowed region* (uncertain region). This model is particularly applicable in scenarios when data points do not lend themselves easily to a single class, as it reduces the inaccuracies in classification by adaptively changing the classification thresholds. game-theoretic shadowed sets achieve this by modeling error management as a strategic game and iteratively optimizing error trade-offs.

4.1 Implementation on Dog Activity Data

In this project, we implement the game-theoretic shadowed sets framework to classify real-world dog activity into three regions: Calm, Active, and Shadowed zone (uncertain). Raw accelerometer values are first normalized to zero mean and unit variance, mitigating inter-day drift and ensuring consistency. Each normalized activity value x is then mapped to a fuzzy membership score $\mu(x)$ using a Gaussian membership function is the following equation [10],

$$\mu_{A_i}(x) \;=\; \text{Gaussian}(x, c_i, \sigma_i) \;=\; \exp\!\left(-\frac{(x - c_i)^2}{2\,\sigma_i^2}\right) \tag{4.1}$$

where

- x is the observed activity level from the collar sensor,
- c_i is the empirical mean of the activity data,
- σ_i is the standard deviation of the activity data.

This mapping produces a continuous score $\mu(x) \in [0, 1]$, with values near 1 indicating vigorous Active movement and values near 0 denoting calm behavior. We initialize thresholds $\alpha_0 = 0.65$ and $\beta_0 = 0.35$ to partition these scores into Active, Calm and Shadowed Zone. Elevation errors (misclassifying calm as active) and reduction errors (misclassifying active as calm) are modeled as two opposing players in a strategic game. In each iteration, α and β are perturbed within $[0, 1]$, misclassification costs are evaluated on the labeled dog data, and payoffs are taken as the negative total error.

4.2 Tradeoff Between Elevation and Reduction Errors

The game-theoretic shadowed sets framework deals with trading off *elevation* and *reduction errors*. These types of errors happen when some data points are moved into full acceptance or full rejection:

- Elevation error: Occurs when membership values are elevated to complete acceptance

(1).

- Reduction error: Occurs when membership values are reduced to complete rejection (0).

These errors need to be minimized, as reducing one tends to increase the other. The game-theoretic shadowed sets framework casts this tradeoff as a competitive game and seeks an equilibrium to balance the two types of errors. Reducing elevation and reduction errors is key to the effectiveness of classification models, more so in the cases of fuzzy sets where data points may not have membership with clear, well-delimited boundaries. The minimization of one type of error will generally maximize the other, hence creating a tradeoff between these two errors. For instance, reducing elevation errors by forcing membership values down may, in turn, increase reduction errors since there will be fewer points accepted into the target class. Likewise, reducing reduction errors by forcing values up may elevate errors, creating false positives.

This complex balance requires careful tuning of fuzzy membership functions, as the functions directly influence the levels of error that are assigned for every data point regarding the target class. As a result of this, the balance between elevation and reduction errors is not a static property; it is actually a dynamic property, which should change based on the interactions of the classification model with new instances and updated thresholds.

4.3 Game-Theoretic Approach to Error Minimization

To address the tradeoff between elevation-reduction error, the game-theoretic shadowed sets framework adopts concepts in game theory, where both types of error are viewed as opposing players.

1. Player E (Elevation Error): Seeks to minimize elevation errors, usually preferring high

threshold values for acceptances.

2. Player R (Reduction Error): Tends to reduce reduction errors, usually preferring smaller values for the non-acceptance threshold.

Each player's strategy then involves adjusting thresholds, based on a *payoff matrix* that quantifies the benefits or losses of each decision. This theoretical framework uses concepts like *Nash equilibrium* and *minimax strategies* to achieve a balanced classification outcome.

In this game-theoretic setting, the strategies of both players are interdependent. A change in one threshold will necessarily affect the other player's performance. For example, increasing the threshold α too high will decrease the elevation error but probably increase the reduction errors since fewer data points are accepted into the target class. Similarly, decreasing the threshold β might decrease reduction errors but increase elevation errors as more points are pushed into full acceptance. The elevated and reduced areas are affected by changes in the thresholds (α, β). The distribution of these areas is significantly influenced by the value of σ, which can be computed using various methods. A value of 0.5 was used by Cattaneo and Ciucci to replace the membership grades of the elements in the shadows [14,15]. The goal is to compromise these conflicting strategies so as to minimize the total error rate for both dimensions. In many practical scenarios, such as pet activity monitoring, minimizing false positives (elevation errors) is often prioritized to avoid unnecessary alerts. The framework supports this by allowing the strategy of Player E to dominate when the cost of incorrect acceptance is higher, leading to a more conservative classification boundary. This explains why, in practice, the game-theoretic shadowed sets model tends to defer uncertain cases rather than classify them as active. The game-theoretic setup, therefore, turns error minimization into a dynamic process through which thresholds can be adjusted adaptively according to the inherent nature of the data.

The solution to this game-theoretic problem is usually solved by optimization techniques, with the equilibrium solution corresponding to those values of α and β that give rise to the minimal total error. One may set this up, for example, in the framework of Nash equilibrium analysis, where the system is iteratively refined such that the strategies of both players converge to an equilibrium that provides an optimal classification boundary. Furthermore, fuzzy membership functions can be incorporated in order to execute the classification process in a much more refined way, making the framework resilient to uncertainty and noise in the data. Hence, the possibility of optimizing the thresholds through game-theoretic approaches is one of the most important advantages in enhancing the effectiveness of the classification models—especially in dealing with data points whose characteristics are not clear or marginal.

4.4 Iterative Learning Mechanism for Threshold Optimization

The game-theoretic shadowed sets framework improves threshold values through a *iterative learning mechanism*. This iterative process is composed of the constant adjustment of thresholds and evaluation of classification effectiveness:

- Initialization: Initialize with extreme threshold values (e.g., $\alpha = 1$ and $\beta = 0$).
- Iterative Adjustments: At each iteration, methods update α and β by evaluating the consequences of classification error.
- Equilibrium and Convergence: Iterations are stopped when thresholds stabilize, reaching an equilibrium where further improvements cannot be made.

During each iteration of learning, the system tests how changing the threshold values (α and β) affects classification errors. It checks whether slightly increasing or decreasing these values helps reduce mistakes. Based on the feedback, the model updates the thresholds to improve its performance. This process continues step by step, always comparing the current accuracy with the previous one to ensure that each change results in improvement. Over time, the adjustments become smaller until the model reaches a stable point called *equilibrium* where further changes no longer improve the results. At this stage, the thresholds are considered optimized, meaning they achieve a good balance between elevation errors (false positives) and reduction errors (false negatives).

This learning process does not assume that the boundary between classes is straight or simple. Instead, it uses fuzzy Gaussian membership functions, which create smooth and flexible decision boundaries that can adapt to complex patterns in the data. To avoid getting stuck in a poor solution (local minimum), the model tries many combinations of thresholds, gradually adjusts values using a decaying learning rate, and may restart from different initial points. These strategies help the system explore better solutions and reach a reliable and accurate result. Overall, this iterative mechanism allows the game-theoretic shadowed sets framework to handle uncertainty, adapt to dynamic data, and find optimal thresholds for robust and accurate classification.

Chapter 5

DESCRIPTION OF IMPLEMENTATION

This chapter describes, step by step, the implementation of the model for pet activity classification. It follows the data preprocessing, model architecture, and training procedures in depth, with a focus on class imbalance handling techniques and model performance improvement. It also covers the used tools and libraries in order to give a wide perspective on how methods have been applied to develop a robust detection system.

5.1 System and Language Specifications

This section explains the computational setup and programming tools employed for the realization of the model. It explains the system environment specifications, from hardware to software configuration. In addition, this section explains the programming language and library selections that were used in building and executing the pet activity classification model.

Computational Environment

The computational environment for this project was based on a HP Pavilion laptop 15-cs0064st, featuring a 8th Generation Intel® Core™ i7 processor with a maximum clock speed of 5.00 GHz, and 8GB of 4800MHz LPDDR4 memory. The processor's high-speed capabilities and multi-core architecture provided the necessary power for handling intensive tasks like training deep learning models. The system also included a 1TB SSD, which facilitated fast data access and reduced the time required for data loading and saving during model training. The laptop ran Windows 11 Home, offering a modern and stable environment compatible with popular machine learning libraries such as NumPy and Matplotlib. This setup allowed for the efficient execution of the pet activity classification model and ensured smooth performance when processing large datasets.

Programming Language Specification

Python selected because it is well-suited for implementing machine learning projects. With Python, large-scale libraries like NumPy and Pandas allow for efficient handling of data in a structured manner, so that the pet activity dataset can be easily processed. Visualization libraries such as Matplotlib were used to visualize the trends present in the data.. Python's ecosystem also supports the evaluation of metrics such as accuracy, precision, recall, and F1-score, which was critical in assessing the performance of the model.

5.2 Libraries and Frameworks

The libraries provide essential functionalities for data manipulation, model building, and evaluation. Their efficient implementations support faster development and enhance the overall performance of the model. Here is a brief explanation of the key Python libraries and modules used in this project [23]:

- **Pandas as pd**

  Pandas is a comprehensive Python library for manipulation and analysis, mainly of structured data like CSV or Excel. This library directly supports loading, preprocessing, and structuring of datasets efficiently to prepare data for analysis. Moreover, Pandas offers powerful tools for exploring data before putting it into an ML model.

- **NumPy as np**

  NumPy is a library that supplies a collection of mathematical functions aimed at array manipulations. It supports enormous multi-dimensional arrays and matrices and provides tools for efficient numerical computations. This functionality is crucial during model development to help optimize calculations.

- **Train-Test Split (from Scikit-Learn)**

  This library is useful to split the dataset into training and testing. the latter can be used to test model performance on unseen data. Testing will, therefore, give an estimation of the generalization capability of the model.

- **Classification Metrics (Scikit-Learn)**

  - **Classification Report:** Summarizes key metrics like accuracy, precision, recall, and F1-score for evaluating model performance.

  - **Accuracy Score:** Provides the ratio of correctly predicted instances to total instances.

5.3 Data Management

Proper data management means accuracy, accessibility, and security are assured at all phases of a datum's life cycle. This section explores strategies and methodologies used in data organization, storage, and processing to enable efficient analysis and decision-making. By laying a foundation for overcoming challenges in data integrity, scalability, and privacy, it ensures the results of using the data are robust and reliable.

Source of Dataset

The dataset used for this project was provided by PetDrifts Company, a pet technology company that specializes in advanced IoT-enabled solutions for monitoring pet health and behavior. In support of this project, PetDrifts facilitated access to a dataset generated by advanced IoT devices installed in pet collars. These collars are equipped with sensors, such as accelerometers and gyroscopes, capable of capturing detailed momentum data, which reflects the intensity of the pet's movement. Each device continuously monitors the activity level of a pet and records the data after fixed intervals, enabling fine-scale monitoring of pet behavior throughout a day. Afterwards, the data was transmitted to a main server or an application, where it prepared the data for the analysis. Besides, the raw data was examined and labeled by expert veterinary behaviorists, who categorized each instance either as 'Calm' or 'Active,' based on the visible pattern within pets' movements; hence, the labels of data are relevant for classification model training.

Dataset Features

The dataset contains several key features that together provide a comprehensive view of the pet's activity patterns. It consists of 3,920 rows, with 1,501 instances labeled as Calm and 2,419 as Active. These data entries represent the activity records of a single dog. Each row corresponds to the average momentum recorded over a 5-minute interval using a wearable collar device. These features include:

- **Pet**: The name of the pet, which is unique in this dataset and serves to distinguish various animals from each other; this enables activity pattern analysis per pet.
- **Timestamp**: Represented in Unix time, this feature gives the exact date and time at which each measurement of pet activity has taken place. This allows for trend and

variation analyses of activity captured over a 5-minute interval.

- **Activity**: A numerical indicator of the pet's movement or activity level at each recorded timestamp. This value probably results from a combination of the raw sensor readings and the result of processing that indicates intensity, including minimal movement to high-intensity activity. This metric is essential for quantifying the activity of the pet. It forms the basis for distinguishing between calm and active states.

- **Status**: A categorical label that identifies the activity state of the pet, as 'Calm' or 'Active'. It was defined by veterinary behaviorists in a subjective manner by analyzing momentum patterns. In this respect, the data is appropriate for supervised learning because a classification model learns from pre-labeled data.

B	C	D	E
Pet	Timestamp	Activity	Status
Alice	1711146390	1.13	Active
Alice	1711146420	0.98	Calm
Alice	1711146450	1	Active
Alice	1711146480	1	Active
Alice	1711146510	1.03	Active
Alice	1711146540	1.02	Active
Alice	1711146570	1.03	Active
Alice	1711146600	1.03	Active
Alice	1711146629	1.06	Active
Alice	1711146659	1.05	Active
Alice	1711146689	1.05	Active
Alice	1711146719	1.21	Active
Alice	1711146749	0.98	Calm
Alice	1711146779	1	Active
Alice	1711146809	1	Active
Alice	1711146839	0.97	Calm
Alice	1711146869	0.99	Calm
Alice	1711146899	0.98	Calm
Alice	1711146929	0.98	Calm
Alice	1711146959	0.98	Calm
Alice	1711146989	0.98	Calm

Figure 5.1: Features of dataset

5.4 Data Preprocessing

The preprocessing phase involved verifying the consistency of activity values by examining their distribution relative to the mean and standard deviation, rather than applying full standardization. To address the class imbalance, synthetic data augmentation was employed using the Synthetic Minority Over-sampling Technique (SMOTE). The original dataset exhibited an underrepresentation of calm instances compared to active ones, which could bias the model during training. SMOTE was used to generate 350 synthetic samples for the calm class by interpolating between existing calm data points and their k-nearest neighbors in the feature space. This process does not duplicate samples but instead creates new, realistic examples within the neighborhood of calm activity patterns. As a result, the total number of calm samples increased to 1,500, which helped to bring the class distribution closer to a 60:40 ratio. This synthetic augmentation improved the model's exposure to calm behavior patterns, leading to more balanced learning and better generalization during classification.Outlier detection and handling were also crucial for improving data quality. Outliers in the dataset were primarily caused by sensor errors. Statistical thresholding methods were applied to identify anomalous data points, which were further validated through visual inspection. The number of outliers detected was relatively small—less than 1% of the total data—and these were manually removed to ensure they did not negatively influence the model's decision boundaries. By eliminating these abnormal entries, the dataset became more consistent and reliable for training the game-theoretic shadowed sets classification model. Collectively, the validation of data range, synthetic augmentation, and outlier filtering ensured the construction of a high-quality which is a critical foundation for robust classification with the game-theoretical shadowed sets model.

5.5 Data Analysis Techniques

The model's generalization ability and efficacy were evaluated using data analysis techniques. This section outlines the methods used to split the dataset to ensure comprehensive model assessment. Techniques like cross-validation were implemented to achieve robust evaluation results.

k-fold Cross-Validation Approach

The stratified k-fold cross-validation technique, with $k = 10$, was used to validate the model's accuracy. In this approach, the dataset was divided into 10 equal parts or folds, and the model was trained and tested iteratively on each fold. For each iteration, one fold served as the test set while the remaining nine folds were used for training, ensuring that each part of the data contributed to both training and testing. This approach provided a more robust and reliable estimate of the performance.

5.6 Model Performance Assessment

The performance of the model was evaluated using the confusion matrix, which provides a detailed breakdown of the model's classification results into four main components: True positives, true negatives, false positives, and false negatives. These values are crucial in calculating key metrics like accuracy, precision, recall, F1-score, which measure the model's ability to identify active and calm classes correctly. Their details are discussed below, based on the model [23]:

Using this confusion matrix, the key metrics like:

1. **Precision:** $\frac{TP}{TP+FP}$ – How many predicted state were actually state.

2. **Recall:** $\frac{TP}{TP+FN}$ – How many actual state were detected.

3. **F1-score:** $2 \times \frac{\text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}}$ – A balanced metric combining precision and recall.

4. **Accuracy:** $\frac{TP+TN}{\text{Total}}$ – Overall correctness of the model.

In summary, the model was evaluated on the test data after training. To assess the predictions, both an accuracy score was used, which gives the overall accuracy of the model, and also a classification report that provided detailed metrics including precision, recall, and F1-score for active and calm, respectively. These provided a sound basis for assessing the model's ability in detecting activity state and performance on this imbalanced dataset.

5.7 Flow Chart: Pet Activity Classification with Game-Theoretic Shadowed Sets

The flowchart presented below illustrates the methodology of the 10-fold cross-validation approach, which was implemented within the framework of game-theoretic shadowed sets specifically designed for the classification of pet activity.

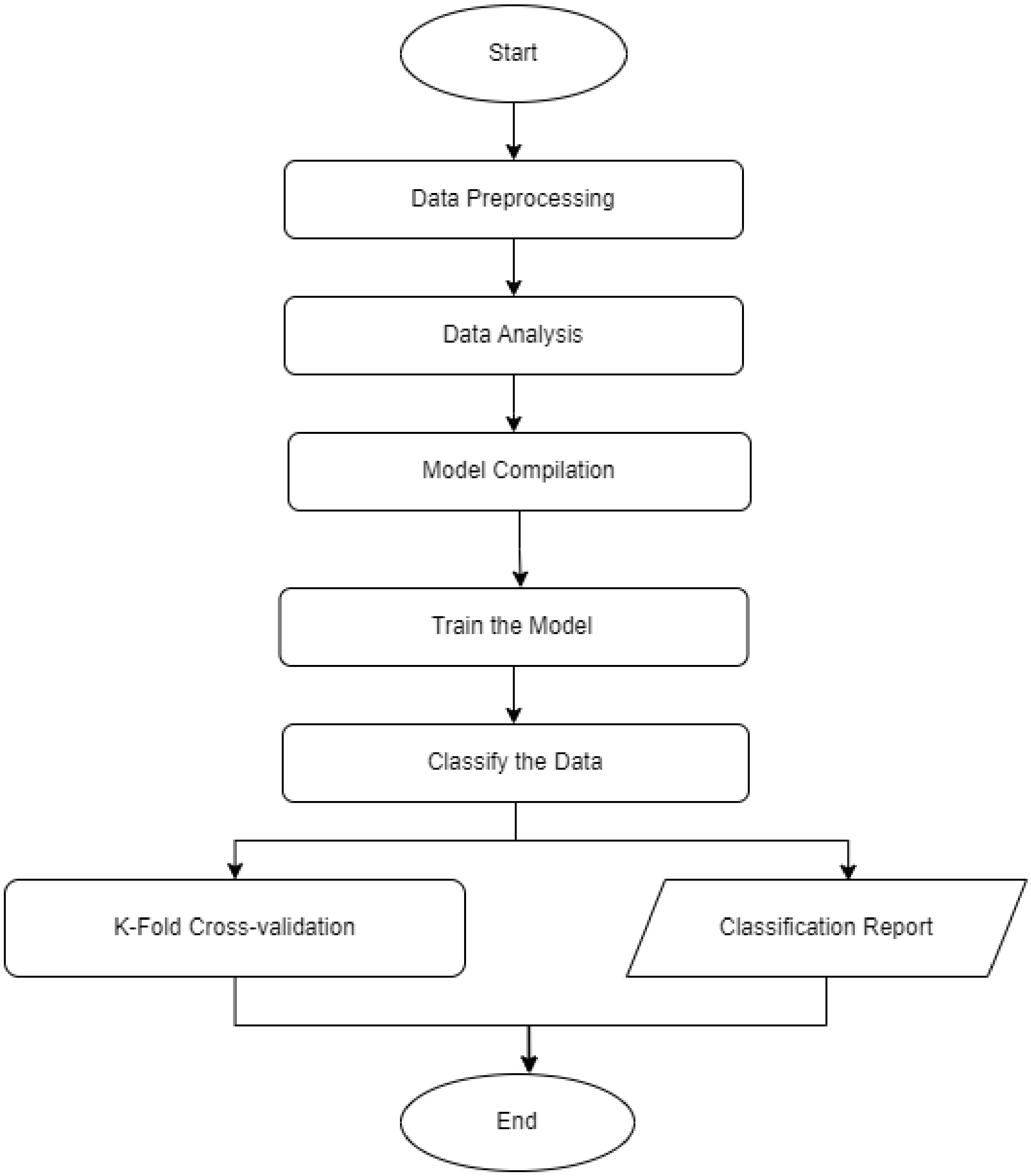

Figure 5.2: Flowchart of the model

Figure 5.2 demonstrates methodology for deployment of a model using K-Fold Cross-Validation. Dataset loading is followed by preprocessing and getting the dataset ready for future analysis. The data is then analyzed to extract insights regarding its features and to verify its suitability for classification operations.

After pre-processing and dataset analysis, K-Fold Cross-Validation is applied to the model to make it robust and prevent overfitting. Here, the training data set is divided into multiple folds and is iteratively trained and tested. Model compilation is first accomplished by initializing and configuring the classification model. Training is accomplished on the training subsets generated by cross-validation. After training, evaluation is accomplished on the model using the test data to make sure that it generalizes to unseen inputs. Then the model is used to classify the data and overall performance is calculated using accuracy, precision, recall, and F1-score.

Finally, a classification report is generated that reflects the overall performance of the model across all validation folds. This structured evaluation ensures a comprehensive assessment of how well the model generalizes to unseen data and provides insightful information for future optimization and tuning.

5.8 Step-by-Step Model Operation

This section is a detailed step-by-step analysis of how the approach game-theoretic shadowed sets is organized into several main phases. These steps include setting up the coding environment, loading and preprocessing the data, applying the Gaussian membership function, adjusting classification thresholds dynamically, classifying the activities, cross-validation methods, and finally visualizing the results.

Step 1: Load Dataset

In this first step, the dataset is loaded into a data frame for its subsequent processing. The dataset is read from a CSV file named `activity_data.csv` using the Pandas `read_csv` function.

```
df = pd.read_csv('activity_data.csv')
```

Step 2: Data Preprocessing

The data preprocessing phase began with the normalization of activity values to ensure consistency across the dataset. Normalization was necessary because the raw activity readings varied in scale and distribution, which could affect the model's ability to interpret them uniformly. Using NumPy, the mean and standard deviation of the activity values were computed, and each data point was rescaled accordingly. This transformation preserved the underlying distribution while aligning the values to a consistent range, allowing for the effective application of the Gaussian membership function during classification.

In addition to normalization, outlier detection was performed to improve data quality and model reliability. Outliers—data points that deviated significantly from the overall distribution—were typically the result of sensor errors. These anomalies were identified using statistical thresholds and confirmed through manual visual inspection. Since the number of such points was minimal (less than 1% of the dataset), they were removed to prevent their influence on model training and decision boundaries.

By completing these preprocessing steps, the dataset was transformed into a clean, balanced, and well-structured input suitable for the classification model. Ensuring high-quality data at this stage was critical, as the effectiveness of the game-theoretic shadowed sets framework depends heavily on the precision and reliability of the input features.

Step 3: Applying the Gaussian Membership Function

The Gaussian membership function forms the backbone of the game-theoretic shadowed sets approach, wherein it transforms normalized activity data into membership scores, quantifying for each activity how well it fits into predefined behavioral categories. This step is important since it provides a base on which subsequent classification will be performed.

This transforms the raw data into a membership score by applying the Gaussian membership function to each value of normalized activity. It refers to the likelihood of a given activity being from, say, ‘Calm’ or ‘Active.’

```
def gaussian_membership(x, c, theta):
    return np.exp(-((x - c) ** 2) / (2 * theta ** 2))
```

Step 4: Dynamic threshold adjustment

The adjustment of dynamic thresholds is one of the most crucial steps in the game-theoretic shadowed sets approach. It fine-tunes the thresholds used for classification based on the inherent nature of the data to reduce the error rate. This is an iterative process that uses the concept of game theory to reach an optimal solution for a balance between different kinds of classification errors.

The classification thresholds were first set through domain knowledge and initial data analysis. A typical threshold might be set at alpha = 0.65 and beta = 0.35, which are the initially set for the boundaries between the ‘Calm’ and ‘Active’ states. In most cases, these a prior threshold values are far from perfect and are usually further iteratively adjusted to fit the data.

It is a dynamic adjustment process in which a number of ‘games’ between the two competing types of classification errors are simulated: elevation errors and reduction errors. Elevation errors occur when a less intense activity is mistakenly classified as more intense, analogous to a Type I error (false positive), where a non-existent effect is incorrectly detected. In contrast, reduction errors occur when a more intense activity is misclassified as less intense, paralleling a Type II error (false negative), where a true effect is missed. This game-theoretic balancing seeks to minimize the overall classification risk by adaptively adjusting thresholds based on the cost and consequence of each error type.

Algorithm:

Algorithm 1 Pet Activity Classification with Game-Theoretic Shadowed Sets Algorithm

Require: Dataset D, initial thresholds α_0, β_0, standard deviation θ, learning rate λ, maximum iterations I_{max}, number of folds k

Ensure: Optimized thresholds α^*, β^*, average accuracy

for each fold in k-fold cross-validation **do**
    Split D into training and testing sets
    Compute mean c and standard deviation θ of training set
    for each x in training set **do**
        Compute membership $\mu = \exp\left(-\frac{(x-c)^2}{2\theta^2}\right)$
    end for
    Initialize $\alpha = \alpha_0$, $\beta = \beta_0$
    for iteration $= 1$ to I_{max} **do**
        for each perturbation d_α, d_β **do**
            Adjust thresholds:
            $\alpha' = \text{clip}(\alpha + d_\alpha, 0, 1)$
            $\beta' = \text{clip}(\beta + d_\beta, 0, \alpha')$
            Compute elevated error E_e and reduction error E_r
            Compute payoff $P = -(E_e + E_r)$
            if P improves **then**
                Update α, β
            end if
        end for
        Decrease λ
        if thresholds stabilized **then**
            break
        end if
    end for
    for each x in test set **do**
        Classify state:
        if $\mu \geq \alpha$ **then**
            State = Active
        else if $\beta < \mu < \alpha$ **then**
            State = Shadowed Zone
        else
            State = Calm
        end if
    end for
    Calculate accuracy for the fold
end for
Compute average accuracy
Visualize membership distribution and thresholds

In each step of the dynamic adjustment process, the model checks the effect of slight adjustments in thresholds. It tests whether an increase or decrease in thresholds will reduce the overall error in classification. This process continues iteratively and gradually refines the thresholds so that the model approaches the optimal classification.

The course of the adjustment is mediated by a learning mechanism that tries to minimize the sum of the elevation and reduction errors. During the process, with every feedback that the model receives from previous iterations, it changes its thresholds until it arrives at those which offer the best compromise between the two types of errors.

The dynamic nature of this process is what makes the game-theoretic shadowed sets approach particularly powerful. Unlike static thresholds, which may not account for the variability and complexity of the data, dynamic thresholds can adjust to the specific characteristics of the dataset, providing a more accurate and nuanced classification.

When this process of dynamic threshold adjustment was completed, the model was then optimized with thresholds fine-tuned toward the data. These formed the threshold basis upon which the classes of activities were finally classified.

```
def adjust_gtss_thresholds(membership, alpha, beta, sigma,
learning_rate, max_iterations):
    previous_alpha, previous_beta = alpha, beta

for iteration in range(max_iterations):
    best_payoff = float('-inf')
    best_alpha, best_beta = alpha, beta

    for d_alpha in np.linspace(-learning_rate, learning_rate, 5):
        for d_beta in np.linspace(-learning_rate, learning_rate, 5):
            new_alpha = np.clip(alpha + d_alpha, 0, 1)
            new_beta = np.clip(beta + d_beta, 0, new_alpha)

            if new_beta >= new_alpha:
                continue

            new_elev_error, new_red_error = calculate_errors(
            new_alpha, new_beta, membership, sigma)
            new_payoff = payoff(new_elev_error, new_red_error)

            if sum(new_payoff) > best_payoff:
                best_payoff = sum(new_payoff)
                best_alpha = new_alpha
                best_beta = new_beta
```

```
# Update alpha and beta to the best found in this iteration
    alpha, beta = best_alpha, best_beta

    if abs(alpha - previous_alpha) < 0.001 and
    abs(beta - previous_beta) < 0.001:
        print(‘Thresholds stabilized, stopping iterations.‘)
        break

    # Update previous values for the next iteration comparison
    previous_alpha, previous_beta = alpha, beta

    # decrease the learning rate to fine-tune the adjustments
    learning_rate *= 0.95

return alpha, beta
```

Step 5: Data Splitting

This step will prepare the data for training and testing using two different approaches: 10-fold cross-validation and analysis of the train-test split ratio. Perform the initialization of k-fold cross-validation with (k=10) splits, while shuffling and setting a fixed random seed.

```
kf = KFold(n_splits=10, shuffle=True, random_state=42)
```

For train-test split ratio analysis, the main elements necessary for data splitting are the input features **X**, representing the independent variables, and the target variable **y**, which becomes the model's prediction. The `train_test_split` function splits the data into training and testing sets; the parameter `test_size=0.20` means that 20% of the data goes for testing and 80% of the data is kept for training. In summary, after the split, **X_train** contains 80% of the feature data for training and **X_test** holds the remaining 20% for evaluation; **y_train** contains 80% of the target labels that correspond to **X_train**, and **y_test** holds the 20% of target labels that correspond to **X_test**.

```
X_train, X_test, y_train, y_test = train_test_split(
    X, y, test_size=0.20, random_state=42)
```

Step 6: Train the Model

The model in this step is trained through 10-fold cross-validation and analysis of the ratio of train-test splits. In every round of 10-fold cross-validation, one subset is used for training while the other is used for validation; class weights will be applied due to the class imbalance problem. For the train-test split, the model is trained on 80% and validated on the remaining 20%. Both models will use early stopping in an effort to prevent overfitting by monitoring their respective validation losses, stopping training if there is no improvement beyond a patience of 10 epochs.

```
# 10-fold Cross-Validation
model.fit(train_data, train_labels, epochs=100, batch_size=256,
verbose=2, validation_data=(test_data, test_labels),
class_weight=class_weights,callbacks=[early_stopping])


# Train-Test Split Ratio Analysis
model.fit(X_train, y_train, epochs=100, batch_size=256, verbose=2,
validation_data=(X_test, y_test), class_weight=class_weights,
callbacks=[early_stopping])
```

Step 7: Evaluate the Model

After training, the model makes predictions on the test data. The function `accuracy_score` calculates the accuracy of the model.

```
average_performance = np.mean(performance_metrics)
print(f'Average Performance (Accuracy): {average_performance}')


std_performance = np.std(performance_metrics)


data.loc[:, 'Membership'] = gaussian_membership(data['Activity'],
c=activity_mean, theta=activity_std)
data['State'] = data['Membership'].apply(lambda x:
classify_activity(x, alpha, beta))
```

Step 8: Completion

Once all folds or the single train-test split are completed, a message is printed to indicate the completion of the training process.

```
print('Training Finished!')
```

Chapter 6

RESULTS AND EVALUATION

Experimental setup, used evaluation metrics, and results obtained by the pet classification model are presented in this section. The experiments were conducted to validate the efficiency of the model and also test the capabilities of the model on challenges including class imbalance, overfitting, and sensitivity for activity state detection. For the model, both train-test split and k-fold cross-validation were used for training and evaluation to make sure that the risk of overfitting is low.

6.1 Results

The effectiveness of the game-theoretic shadowed sets framework was evaluated using 10-Fold Cross-Validation. This method ensures robust performance estimation by dividing the dataset into ten equally sized folds. The model was trained on nine folds and tested on the remaining fold, repeating this process for each fold. The accuracy values for each fold were calculated, and the average accuracy was used to summarize the overall model performance.

Accuracy is defined here as the number of correctly classified instances divided by the total number of instances in the test set for each fold. The average accuracy across all folds is taken into account to give a good measure for performance.

The results of 10-fold cross-validation demonstrate excellent and consistent performance of the proposed game-theoretic shadowed sets model, achieving an average accuracy of 91.17%. This study focuses solely on evaluating game-theoretic shadowed sets and does not include experimental comparisons with other classifiers such as decision trees, support vector machines, or neural networks. Table 6.1 describes the obtained accuracy for each fold.

Table 6.1: Fold-wise Performance Metrics of the GTSS Model

Fold	**Accuracy**	**Precision**	**Recall**	**F1 Score**
1	87.44%	92.1%	83.3%	87.49%
2	99.59%	98.5%	99.8%	99.15%
3	97.97%	96.9%	98.8%	97.85%
4	98.78%	99.5%	98.1%	98.79%
5	87.04%	85.2%	89.1%	87.10%
6	63.96%	70.2%	64.3%	67.12%
7	91.09%	87.64%	98.69%	92.83%
8	98.78%	97.6%	99.4%	98.48%
9	99.19%	99.4%	98.7%	99.05%
10	87.87%	88.9%	86.3%	87.59%
Average	**91.17%**	**91.59%**	**91.64%**	**91.54%**

The game-theoretical shadowed sets model achieved an average accuracy of 91.17% across all folds, demonstrating its overall effectiveness in classifying pet activity states. While one fold exhibited relatively lower accuracy, the remaining folds showed minimal variation, indicating the model's stability and strong generalization capability. These results collectively validate the reliability and robustness of the game-theoretical shadowed sets framework for accurately distinguishing between calm and active pet behaviors in real-world scenarios.

The classification performance of the proposed game-theoretical shadowed sets model was further evaluated using a confusion matrix, which compares predicted and actual activity states—Calm and Active. The model accurately identified 260 out of 324 Calm instances, yielding a recall of approximately 80.25% and a very high precision of 97.74% for the Calm class. In contrast, the model correctly detected 454 out of 460 Active instances, achieving an

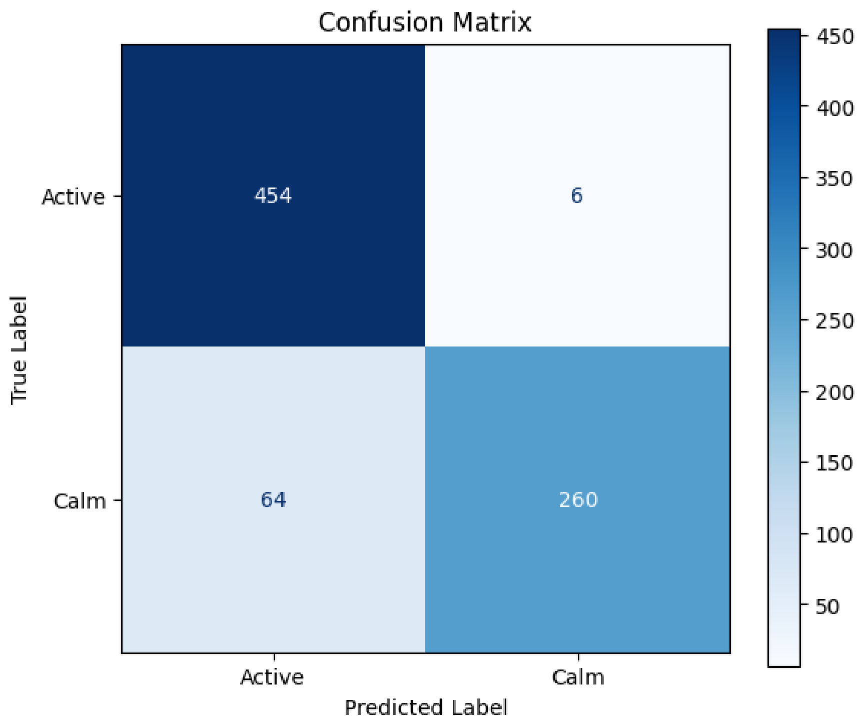

Figure 6.1: Confusion matrix showing classification performance of the GTSS model.

excellent recall of 98.69% and a precision of 87.64% for the Active class. This classification pattern illustrates the conservative nature of game-theoretical shadowed sets, which prioritizes minimizing false positives, particularly for the Active class, by deferring ambiguous cases. Overall, the confusion matrix confirms that the game-theoretical shadowed sets model offers high classification accuracy and effective uncertainty handling, making it well-suited for real-world pet monitoring applications where minimizing false alerts is essential.

6.2 Applications in Veterinary Science and Pet Care

The purpose of this section is to illustrate how the proposed pet activity classification model could be useful in real-world veterinary and pet care contexts. While the current system is not designed to directly interface with veterinary diagnostics, it can serve as an early warning tool that highlights unusual activity trends. Such patterns may prompt further investigation or veterinary consultation. The examples provided here are illustrative in nature and are intended to make the potential applications more relatable and easier to understand for general readers.

Pet Health Monitoring

Monitoring a dog's activity levels is crucial for detecting early signs of illness or discomfort. For instance, if a typically energetic dog, such as a Labrador Retriever, suddenly spends excessive time in a calm or sleep state, it could indicate health issues like joint pain, muscle soreness, or digestive problems [12]. Older dogs, prone to conditions like arthritis, may naturally reduce their movement to minimize discomfort. The momentum-based activity classifier aids in identifying these subtle changes promptly, allowing for early intervention.

Example: A seven-year-old Golden Retriever named Max exhibited a noticeable decrease in his daily activity levels over a week. Using the classifier, his owner detected this change early and consulted a veterinarian. Max was diagnosed with early-stage arthritis, and timely intervention with medication and physical therapy improved his condition, enhancing his quality of life.

Environmental factors also play a significant role in a dog's activity levels. Changes such as hot weather can lead to lethargy, while excessive noise might increase stress levels, which can alter activity patterns [38]. For instance, a dog disturbed by nearby construction noise may exhibit increased movement or frequent interruptions during rest. Analyzing these patterns enables owners to make environmental or routine adjustments to improve their dog's comfort and reduce stress.

Seasonal health impacts are another area where the classifier proves beneficial. Dogs may exhibit decreased activity during colder months due to unfavorable weather conditions or even seasonal affective disorder [29]. Through long-term data analysis, owners can recognize these patterns and adjust care routines accordingly, such as incorporating more indoor activities during winter.

Sleep Quality Assessment

Quality sleep is essential for a dog's health and well-being. The classifier differentiates between deep, uninterrupted sleep and fragmented sleep characterized by frequent movements [11]. Dogs that shift often during sleep may be experiencing discomfort, pain, or anxiety. Identifying such patterns allows owners to investigate potential causes, like uncomfortable bedding or underlying health issues, and take corrective actions.

Example: Bella, a five-year-old Beagle, began experiencing fragmented sleep as detected by the classifier. Upon further investigation, her owner discovered that a new heating system was causing the bedroom to become too warm at night. After adjusting the temperature, Bella's deep sleep patterns were restored, improving her rest quality.

External disturbances, such as bright lights, temperature fluctuations, or nighttime noises, can also disrupt a dog's sleep [44]. By comparing sleep data before and after making environmental adjustments, owners can pinpoint and mitigate factors causing restless nights. This is particularly important for aging dogs, which may suffer from conditions like canine cognitive dysfunction (CCD), leading to sleep disruptions that require specific management strategies [27].

Behavioral Analysis and Training

Understanding a dog's engagement levels is essential for effective training. The classifier provides insights into how long a dog remains active or engaged during training sessions [13]. If a dog loses focus quickly, trainers can adapt their methods by incorporating shorter sessions or using high-energy rewards to maintain attention, enhancing the training's effectiveness.

Example: A Border Collie named Luna struggled with extended training sessions, often losing focus after 15 minutes. By monitoring her activity levels, the trainer adjusted the sessions to shorter, more engaging periods, resulting in improved obedience and enthusiasm from Luna.

Data on calm and active periods also help determine a dog's endurance and attention span. While a young, high-energy breed may sustain longer active states, older or less energetic breeds might require more frequent breaks [21]. Tracking progress over time is invaluable, especially for shy or anxious dogs, as gradual increases in active time can indicate growing confidence and improved focus.

Exercise and Weight Management

Regular exercise is essential for a dog's physical health and weight management. Each breed has specific exercise requirements. For instance, an active breed like a Siberian Husky may require one to two hours of vigorous activity daily, whereas a Pug might benefit from shorter, less intense sessions due to their brachycephalic nature [40].

Table 6.2: Exercise Requirements by Breed

Breed	**Exercise Classification**	**Recommended Daily Exercise**
Border Collie	High Energy	1.5 - 2 hours
Labrador Retriever	High Energy	1 - 1.5 hours
Beagle	Moderate Energy	1 hour
Bulldog	Low Energy	30 minutes
German Shepherd	High Energy	1 - 2 hours
Chihuahua	Low to Moderate Energy	20 - 30 minutes
Golden Retriever	High Energy	1 - 1.5 hours
Dachshund	Low to Moderate Energy	30 - 45 minutes
Siberian Husky	Very High Energy	2 hours
Great Dane	Moderate Energy	30 - 45 minutes
Shih Tzu	Low Energy	20 - 30 minutes
Jack Russell Terrier	Very High Energy	1 - 1.5 hours
Pug	Low to Moderate Energy	30 minutes
Rottweiler	Moderate to High Energy	1 - 1.5 hours
Poodle (Standard)	High Energy	1 - 1.5 hours

Monitoring activity levels helps prevent obesity and associated health issues, particularly in breeds prone to weight gain, such as Beagles and Dachshunds [19]. If a dog's activity falls short of its daily requirement, owners can adjust by increasing playtime or walks. Additionally, for dogs with physical conditions like hip dysplasia, the classifier aids in identifying suitable low-impact exercises that maintain fitness without causing strain [17].

Anxiety and Mental Health Observation

Behavioral issues such as separation anxiety can be challenging to detect without concrete data [45]. An increase in active time when the owner is absent may indicate anxiety. If a dog's rest or sleep time is disrupted only when alone, the classifier provides evidence for this diagnosis, enabling owners to seek appropriate behavioral interventions.

Example: Charlie, a two-year-old French Bulldog, showed increased activity levels during his owner's absence, as recorded by the classifier. Recognizing this pattern, the owner consulted a behaviorist, and with targeted training and environmental enrichment, Charlie's separation anxiety symptoms improved over time.

Nocturnal restlessness is another area of concern. Increased activity during the night may signal anxiety, discomfort, or sleep disorders [20]. This could be due to unknown sounds, night terrors, or an inability to settle down. Recognizing these patterns allows for timely interventions, such as creating a more conducive sleeping environment or consulting a veterinarian for further assessment.

Medical Diagnostics Support

The classifier serves as a supportive tool in medical diagnostics. For dogs suspected of arthritis, trends showing reduced active periods or increased calm states can prompt veterinary consultations for further evaluation [25]. Early detection allows for timely interventions, like medications or therapies, to improve mobility and quality of life.

Hormonal disorders, such as Cushing's disease, often manifest through restlessness and unusual activity spikes, particularly at night [18]. If the classifier detects such patterns, it can alert owners and veterinarians to conduct appropriate hormonal tests. Similarly, monitoring cognitive dysfunction in older dogs is crucial. Erratic sleep-wake cycles and frequent nighttime activity can support a diagnosis of canine cognitive dysfunction, guiding the development of a comprehensive care plan [47].

Potential Outcomes of Using the Classifier

The results from the activity classifier can help people take useful actions, such as spotting unusual behavior or adjusting the dog's care routine. The output of the system is a detailed daily or weekly summary of the dog's activity, breaking it down into time spent in calm, and active states. This at-a-glance view helps owners understand their dog's typical energy levels and quickly identify any deviations from the norm.

Long-term data collection will allow one to observe behavioral trends with the influencing seasonal and environmental factors. Pet owners might notice, for example, that their dog is less active in winter or becomes restless when there is stress at home. The system can also be set to send auto-alerts in cases of serious changes in activity patterns.

Analysis of the balance between rest and activity offers insights into the life style of the dog in general. The classifier assists in ensuring the dog maintains a healthy balance, and adjustments can be made for dogs recovering from illness or injury. Furthermore, the classifier evaluates sleep quality by monitoring periods of deep versus light sleep, noting interruptions. Identifying changes in sleep quality allows owners to address potential problems by changing conditions of sleep or seeking veterinary advice.

Chapter 7

CONCLUSION

The project of implementing game-theoretic shadowed sets for pet activity classification underlines the transformative potential of integrating advanced mathematical models with behavioral analysis. Dynamic adaptability assured by game-theoretic principles positions this model as a breakthrough tool for real-world applications. The ability to classify such activities in a nuanced manner-calm, active, and transitional-enables pet owners and veterinarians to monitor and manage animal well-being with accuracy. By dynamically adjusting classification thresholds, the game-theoretic shadowed sets accommodates variability in pet behavior, ensuring accurate and meaningful insights. This adaptability is particularly beneficial for long-term behavioral monitoring, allowing for timely interventions and improved pet health outcomes.

The game-theoretic shadowed sets approach has been successful with its comprehensive methodology that ranges from robust data collection and detailed labeling to sophisticated analytic processes, including the implementation of a Gaussian membership function and dynamic threshold adjustment. Every part of the methodology synergizes to make a model that will be able to tell something about complex behavioral patterns in pets. These will also go beyond academic insights into practical applications in veterinary diagnostics, behavioral training, and customized solutions for pet care. For example, veterinarians may use the outputs of the model to identify health concerns, while the owners will be able to create better routines for care upon noticing specific patterns of activity.

While it is an innovative piece, this technology can be further developed. Improvements in the system could come by incorporating more data sources, including heart rate monitors or environmental sensors, to further improve classification accuracy and behavioral insights. Additionally, a major direction for future research is to conduct a comparative evaluation between the game-theoretic shadowed sets framework and more traditional or simpler classification models, such as decision trees, support vector machines, or neural networks. This comparison would help assess whether the added complexity of game-theoretic shadowed sets provides significant advantages over simpler methods in real-world scenarios. Furthermore, adapting the game-theoretic shadowed sets framework for other species or broader behavioral contexts can extend its usefulness across various domains. This work not only contributes to the field of pet activity classification but also paves the way for using advanced decision-theoretic models to improve animal care and welfare. The game-theoretic shadowed sets framework will bridge the gap between theoretical research and practical applications, showing how computational tools can be utilized to support healthier, happier lives for pets and their owners.

REFERENCES

[1] T. Camps, M. Amat, X. Manteca: A review of medical conditions and behavioral problems in dogs and cats, Animals **9**(12), 1133 (2019).

[2] K. Tiira, E. Sulkama, H. Lohi: Prevalence, comorbidity, and behavioral variation in canine anxiety, Scientific Reports **6**, 39479 (2016).

[3] S. Hoppe, D. Fäh, H. Reusch: Correlates of attention deficit hyperactivity disorder (ADHD)-like behavior in domestic dogs, Journal of Veterinary Behavior **22**, 46–54 (2017).

[4] P. Kumpulainen, A. Valldeoriola Card´o, S. Somppi, H. Törnqvist, H. Väätäjä, P. Majaranta, Y. Gizatdinova, C. Hoog Antink, V. Surakka, M.V. Kujala, O. Vainio, A. Vehkaoja: Dog behaviour classification with movement sensors placed on the harness and the collar, Applied Animal Behaviour Science **241**, 105393 (2021).

[5] D.S. Mills, I. Demontigny-Bédard, M. Gruen, M.P. Klinck, K.J. McPeake, A.M. Barcelos, L. Hewison, H. Van Haevermaet, S. Denenberg, H. Hauser, C. Koch, K. Ballantyne, C. Wilson, C.V. Mathkari, J. Pounder, E. Garcia, P. Darder, J. Fatj´o, E. Levine: Pain and problem behavior in cats and dogs, Animals **10**(2), 318 (2020).

[6] P. Kasnesis, V. Doulgerakis, D. Uzunidis, D.G. Kogias, S.I. Funcia, M.B. González, C. Giannousis, C.Z. Patrikakis: Deep learning empowered wearable-based behavior recognition for search and rescue dogs, Sensors **22**(3), 993 (2022).

[7] H. Wang, O. Atif, J. Tian, J. Lee, D. Park, Y. Chung: Multi-level hierarchical complex behavior monitoring system for dog psychological separation anxiety symptoms, Sensors **22**(4), 1556 (2022).

[8] J.J. Valletta, C. Torney, M. Kings, A. Thornton, J. Madden: Applications of machine learning in animal behaviour studies, Animal Behaviour **124**, 203–220 (2017).

[9] J.P. Herbert, J.T. Yao: Game-theoretic rough sets, Fundamenta Informaticae **108**(3–4), 267–286 (2011).

[10] A.J. Albrecht, J.E. Gaffney: Software function, source lines of code, and development effort prediction: a software science validation, IEEE Transactions on Software Engineering **9**(6), 639–648 (1983).

[11] G.J. Adams, K.G. Johnson: Sleep, work, and the effects of shift work in drug detector dogs canis familiaris, Applied Animal Behaviour Science **36**(1), 1–9 (1993).

[12] D. C. Brown, R. C. Boston, J. C. Coyne, J. T. Farrar: Ability of the canine brief pain inventory to detect response to treatment in dogs with osteoarthritis, Journal of the American Veterinary Medical Association **233**(8), 1278–1283 (2010).

[13] C.L. Battaglia: Periods of early development and the effects of stimulation and social experiences in the canine, Journal of Veterinary Behavior **4**(5), 203–210 (2009).

[14] G. Cattaneo, D. Ciucci: An algebraic approach to shadowed sets, Electronic Notes in Theoretical Computer Science **82**(4), 64–75 (2003).

[15] G. Cattaneo, D. Ciucci: Shadowed sets and related algebraic structures, Fundamenta Informaticae **55**(3–4), 255–284 (2003).

[16] L. Chen, J. Zou, C.L.P. Chen: Kernel spatial shadowed c-means for image segmentation, International Journal of Fuzzy Systems **16**(1), 46–56 (2016).

[17] J.B. Van Dyke, M.C. Zink: Canine sports medicine and rehabilitation, John Wiley and Sons pp. 201–222 (2013).

[18] E.C. Feldman, R.W. Nelson: Canine and feline endocrinology and reproduction, Elsevier Health Sciences, 2004.

[19] A.J. German: The growing problem of obesity in dogs and cats, The Journal of Nutrition **136**(7), 1940S–1946S (2006).

[20] L. Gruen, Entangled Empathy: An alternative ethic for our relationships with animals, Lantern Books, New York (2015).

[21] A. Haverbeke, B. Laporte, E. Depiereux, J. M. Giffroy, C. Diederich: Training methods of military dog handlers and their effects on the team's performances, Applied Animal Behaviour Science **113**(1–3), 110–122 (2008).

[22] C. -T. Huang, M. N. Sakib, C. Kamhoua, K. Kwiat, L. Njilla: A game theoretic approach for inspecting web-based malvertising, IEEE International Conference on Communications (ICC), Paris, France, 1–6 (2017). doi: 10.1109/ICC.2017.7996807.

[23] H. He, E.A. Garcia: Learning from imbalanced data, IEEE Transactions on Knowledge and Data Engineering **21**(9), 1263–1284 (2009).

[24] O. Hryniewicz: Bayes statistical decisions with random fuzzy data: an application in reliability, Reliability Engineering and System Safety **151**, 20–33 (2016).

[25] S.A. Johnston: Osteoarthritis. Joint anatomy, physiology, and pathobiology, Veterinary Clinics of North America: Small Animal Practice **27**(4), 699–723 (1997).

[26] C.M. Jiang, Z.C. Li, J.T. Yao: A shadowed set-based three-way clustering ensemble approach, International Journal of Machine Learning and Cybernetics, **13**(9), 2545–2558, (2022).

[27] G.M. Landsberg, W.L. Hunthausen, L.J. Ackerman: Behavior problems of the dog and cat, Elsevier Health Sciences, (2012).

[28] X.L. Li, P. Geng, B.Z. Qiu: A cluster boundary detection algorithm based on shadowed set, Intelligent Data Analysis **20**(1), 29–45 (2016).

[29] I. Meyer, B. Forkman: Dog and owner characteristics affecting the dog-owner relationship, Journal of Veterinary Behavior **9**(4), 143–150 (2014).

[30] C. Ladha, N. Hammerla, E. Hughes, P. Olivier, P. Ploetz, Dog's life: wearable activity recognition for dogs, in Proceedings of the ACM Conference on Pervasive and Ubiquitous Computing Adjunct (UbiComp '13 Adjunct), 415–418 (2013).

[31] T. Kiyohara, R. Orihara, Y. Sei, Y. Tahara, A. Ohsuga: Activity recognition for dogs based on time-series data analysis, in B. Duval, J. van den Herik, S. Loiseau, J. Filipe (eds.): Agents and artificial intelligence, ICAART 2015, Lecture Notes in Computer Science **9494**, Springer (2015).

[32] H. Wang, O. Atif, J. Lee, D. Park, Y. Chung: Sensor-based dog abnormal behavior detection (2019).

[33] A. Hussain, A. Shigri, M. Abdullah, H.C. Kim: Activity detection for the wellbeing of dogs using wearable sensors based on deep learning, IEEE Access (2022).

[34] S. Aich, S. Chakraborty, J.-S. Sim, D.-J. Jang, H.-C. Kim: The design of an automated system for the analysis of the activity and emotional patterns of dogs with wearable sensors using machine learning, Applied Sciences **9**(22), 4938 (2019).

[35] S. Mitra, P.P. Kundu: Satellite image segmentation with shadowed c-means, Information Sciences **181**(17), 3601–3613 (2011).

[36] J. von Neumann, O. Morgenstern: Theory of games and economic behavior. Princeton University Press, Princeton (1944).

[37] J.F. Nash: Equilibrium points in n-person games, Proceedings of the National Academy of Sciences of the United States of America **36**(1), 48–49 (1950).

[38] K.L. Overall: Manual of clinical behavioral medicine for dogs and cats. Elsevier Health Sciences, Amsterdam (2013).

[39] M.J. Osborne, A. Rubinstein: A course in game theory. MIT Press, Cambridge (1994).

[40] R.M. Packer, A. Hendricks, C.C. Burn: Impact of facial conformation on canine health: corneal ulceration, PLoS ONE **7**(4), e33917 (2012).

[41] W. Pedrycz: Shadowed sets: representing and processing fuzzy sets, IEEE Transactions on Systems, Man, and Cybernetics - Part B: Cybernetics **28**(1), 103–109 (1998).

[42] W. Pedrycz, G. Vukovich: Granular computing with shadowed sets, International Journal of Intelligent Systems **17**(2), 173–197 (2002).

[43] W. Pedrycz: Interpretation of clusters in the framework of shadowed sets, Pattern Recognition Letters **26**(15), 2439–2449 (2005).

[44] N. Rooney, S. Gaines, E. Hiby: A practitioner's guide to working dog welfare, Journal of Veterinary Behavior **4**(3), 127–134 (2009).

[45] B.L. Sherman, D.S. Mills: Canine anxiety and phobias: an update on separation anxiety and noise aversions, Veterinary Clinics of North America: Small Animal Practice **38**(5), 1081–1106 (2008).

[46] L.S. Shapley: A value for n-person games, Annals of Mathematics Studies **28**, 307–317 (1953).

[47] T. Schütt, N. Toft, M. Berendt: Cognitive function, progression of age-related behavioral changes, biomarkers, and survival in dogs more than 8 years old, Journal of Veterinary Internal Medicine **29**(6), 1569–1577 (2015).

[48] Y. Shoham, K. Leyton-Brown: Multiagent systems: algorithmic, game-theoretic, and logical foundations. Cambridge University Press, Cambridge (2009).

[49] J.T. Yao, A.V. Vasilakos, W. Pedrycz: Granular computing: perspectives and challenges, IEEE Transactions on Cybernetics **43**(6), 1977–1989 (2013).

[50] S.M. Zabihi, M.-R. Akbarzadeh-T: Generalized fuzzy c-means clustering with improved fuzzy partitions and shadowed sets, ISRN Artificial Intelligence (2012), 1–6.

[51] J. Zhou, Z.H. Lai, D.Q. Miao, C. Gao, X.D. Yue: Multigranulation rough-fuzzy clustering based on shadowed sets, Information Sciences (2018).

[52] Y. Zhou, H.B. Su, H.T. Zhang: A novel data selection method based on shadowed sets, Procedia Engineering **15**, 1410–1415 (2011).

[53] Y. Zhang, J.T. Yao: Game theoretic approach to shadowed sets: a three-way tradeoff perspective, Information Sciences **507**, 540–552 (2020).

[54] Y. Zhang, J.T. Yao: Determining Strategies in Game-Theoretic Shadowed Sets, Information Processing and Management of Uncertainty in Knowledge-Based Systems. Theory and Foundations, Communications in Computer and Information Science **854**, 736–747 (2018).

[55] Y. Zhang, Y. Zhou, J.T. Yao: Feature Extraction with TF-IDF and Game-Theoretic Shadowed Sets, Information Processing and Management of Uncertainty in Knowledge-Based Systems. Theory and Foundations, Communications in Computer and Information Science **1237**, 722–733 (2020).