Granular Computing and Sequential Three-Way Decisions

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Abstract. Real-world decision making typically involves the three options of acceptance, rejection and non-commitment. Three-way decisions can be motivated, interpreted and implemented based on the notion of information granularity. With coarse-grained granules, it may only be possible to make a definite decision of acceptance or rejection for some objects. A lack of detailed information may make a definite decision impossible for some other objects, and hence the third non-commitment option is used. Objects with a non-commitment decision may be further investigated by using fine-grained granules. In this way, multiple levels of granularity lead naturally to sequential three-way decisions.[1]

1 Introduction

Two fundamental notions of rough set theory are knowledge granularity [15, 16] and the approximation of a concept by a pair of lower and upper approximations [3, 4] or three regions. In this paper, I argue that the two notions play an equally important role in a theory of three-way decisions [29]. Three-way decisions can be motivated, interpreted and implemented based on the notion of information and knowledge granularity. Three regions of rough sets [15], and in particular probabilistic rough sets [3, 4, 24, 25], lead naturally to three-way decisions [27, 28], which may produce better results in rule learning [5]. A theory of three-way decisions may be viewed an extension of rough set theory, based on the same philosophy but goes beyond. Three-way decisions focus on a more general class of problems where a set of objects are divided into three pair-wise disjoint regions [2, 29].

A two-way decision consists of either an acceptance or a rejection of an object for a specific purpose. However, a two-way decision may not always be possible in real life in the context of multiple levels of granularity and multiple levels of approximations. At a higher level of granularity, one may have a more

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abstract and compact representation of a decision problem by omitting details, leading to a faster decision process but a less accurate result. On the other hand, at a lower level of granularity, one may have a more concrete and elaborate representation, leading to a slower decision process but a more accurate result. Therefore, making the right decision at the right level is a crucial issue. Three-way decisions, consisting of acceptance, rejection, and non-commitment, are a practical solution. When the available information is insufficient or the evidence is not strong enough to support an acceptance or a rejection at a particular level of granularity, a third option of non-commitment allows us to defer a decision to the next level of granularity.

Three-way decisions may be related to a basic principle of granular computing. By utilizing granular structures, granular computing [1, 17, 21, 22, 31] focuses on a set of philosophy, methodology and paradigm for structured thinking, structured problem solving and structured information processing at multiple levels of granularity [26]. Granular structures consist of many hierarchies for multiview descriptions of a problem, with each hierarchy being composed of multiple levels of abstraction [26]. In an earlier paper [23], I stated that a basic principle of computing, guided by granular structures, is to

> *"· · · examine the problem at a finer granulation level with more detailed information when there is a need or benefit for doing so."*

The objective of the present study is to introduce sequential three-way decisions based on this principle. We want to make a decision "at a finer granulation level with more detailed information when there is a need or benefit for doing so."

There are two contributions from the study. One is to provide a granular computing perspective on three-way decisions. I will demonstrate that three-way decisions are superior and necessary in the context of multiple levels of information granularity. That is, a decision problem is more appropriately formulated as a sequence of three-way decisions, leading finally to two-way decisions. The other is a demonstration of a basic principle of granular computing and, hence, makes it easily understandable and applicable to a wide range of applications.

2 An Overview of Three-way Decisions

By extending the three-way classification of rough set theory and synthesizing results across many disciplines, I examined a theory of three-way decisions in an earlier paper [29]. The main results are briefly reviewed in this section.

Suppose U is a finite nonempty set of objects and $\mathbf{C}$ is a finite set of conditions. Depending on applications, a condition in $\mathbf{C}$ may be a criterion, an objective, or a constraint. A decision task is to divide U into regions according to the satisfiability of objects of the set of conditions $\mathbf{C}$. Formally, **the problem of three-way decisions** can be stated as follows:

> The problem of three-way decisions is to divide U, based on the set of conditions $\mathbf{C}$, into three pair-wise disjoint regions, $\mathrm{POS}, \mathrm{NEG}$, and BND,

called the positive, negative, and boundary regions, respectively. The positive region POS consists of those objects that we *accept* as satisfying the conditions and the negative region NEG consists of those objects that we *reject* as satisfying the conditions. For objects in the boundary region BND, we neither accept nor reject, corresponding to a non-commitment.

The satisfiability reflects a nature of the objects. It may be either qualitative or quantitative; it may also be known, partially known, or unknown. For an object $x \in U$, let $s(x)$ denote the satisfiability of x of the set of conditions $\mathbf{C}$ and is called the state of x. Depending on the set of all possible values of $s(\cdot)$, we may have two-state and many-state decisions problems.

For the two-state case, if we know the true state $s(x)$ for every object, we do not really need three-way decisions, as we can simply classify objects into two regions based on $s(x)$. In many situations, we may not know the true state of an object and may only construct a function $v(x)$ to help us in probing the true state $s(x)$. The value $v(x)$ is called the decision status value of x and may be interpreted as the probability or possibility that x satisfies $\mathbf{C}$. In this context, three-way decisions seem to be appropriate. For the many-state case, even if we know $s(x)$, a three-way decision is still necessary. The results of three-way decisions may be viewed as a three-valued approximation.

In the rest of this paper, I only consider a two-state three-way decisions model that uses an evaluation $v : U \longrightarrow L$ to estimate the states of objects in U, where $(L, \preceq)$ is a totally ordered set. By introducing a pair of thresholds (α, β), $\beta \prec \alpha$ (i.e., $\beta \preceq \alpha$ and $\beta \neq \alpha$), on the evaluation v, we construct three regions as follows:

$$\begin{aligned}
\mathrm{POS}_{(\alpha,\beta)}(v) &= \{x \in U \mid v(x) \succeq \alpha\},\\
\mathrm{NEG}_{(\alpha,\beta)}(v) &= \{x \in U \mid v(x) \preceq \beta\},\\
\mathrm{BND}_{(\alpha,\beta)}(v) &= \{x \in U \mid \beta \prec v(x) \prec \alpha\},
\end{aligned} \tag{1}$$

where for $a, b \in L$, $a \succeq b \Longleftrightarrow b \preceq a$ and $a \prec b \Longleftrightarrow (a \preceq b, a \neq b)$. Condition $\beta \prec \alpha$ implies that the three regions are pair-wise disjoint. Since some of the regions may be empty, the three regions do not necessarily form a partition of the universe U.

From the formulation, we must consider at least the following issues:

– Construction and interpretation of the totally ordered set $(L, \preceq)$.
– Construction and interpretation of the evaluation $v(\cdot)$.
– Construction and interpretation of the pair of thresholds (α, β).

The value $v(x)$ may be interpreted as the probability, possibility or degree to which x satisfies $\mathbf{C}$. The pair of threshold (α, β) can be related to the cost or error of decisions. Those notions will be further discussed in the next section.

3 A Model of Sequential Three-way Decisions

In this section, I propose a sequential three-way decision model for two-state decision model and show its advantages over two-way decisions.

3.1 Simple Two-way Decisions

For a two-state decision problem, we assume that each object $x \in U$ is in one of the two states: either satisfies the set of conditions $\mathbf{C}$ or does not. The state of an object is an inherent property of the object, independent of whether we have sufficient information to determine it. Let a mapping $s : U \longrightarrow \{0, 1\}$ denote the states of all objects as follows:

$$s(x) = \begin{cases} 1, & x \text{ satisfies } \mathbf{C}, \\ 0, & x \text{ does not satisfy } \mathbf{C}. \end{cases} \tag{2}$$

We make a decision regarding the true state of an object based on a representation, a description of, or some information about x. In many situations, the available information may be incomplete and uncertain, and the set of conditions may not be formally and precisely stated. It is impossible to determine the state of each object with certainty. We can construct an evaluation function to assist in a decision-making process.

Let $\mathrm{Des}(x)$ denote a description of x and U_D denote the set of all possible descriptions. An evaluation, $v : U_D \longrightarrow L$, is now given by a mapping from U_D to a totally ordered set $(L, \preceq)$. The quantity $v(\mathrm{Des}(x))$ is called the decision status value of x. Intuitively, a larger value $v(\mathrm{Des}(x))$ suggests that the object x satisfies the conditions $\mathbf{C}$ to a higher degree. Based on the decision status values and a threshold $\gamma \in L$, we can divide U into a positive region and a negative region based on a strategy of two-way decisions:

$$\begin{aligned} \mathrm{POS}_\gamma(v) &= \{x \in U \mid v(\mathrm{Des}(x)) \succeq \gamma\}, \\ \mathrm{NEG}_\gamma(v) &= \{x \in U \mid v(\mathrm{Des}(x)) \prec \gamma\}. \end{aligned} \tag{3}$$

The positive region consists of those objects that we *accept* as satisfying the conditions in $\mathbf{C}$ and negative region consists of those objects that we *reject* as satisfying the conditions in $\mathbf{C}$.

3.2 Sequential Three-way Decisions

In the simple two-way decisions, we use a single representation of an object. In real-world decision making, we may consider a sequence of three-way decisions that eventually leads to two-way decisions. At each stage, new and more information is acquired. For example, in clinical decision making, based on available information, a doctor may decide to treat or not to treat some patients; for some other patients, the doctor may prescribe further tests and defer a decision to the next stage [14]. The basic ideas of sequential three-way decisions appear in a model of sequential three-way hypothesis testing introduced by Wald [20] and a model of sequential three-way decisions with probabilistic rough sets [30]. Li et al. [7] consider a sequential strategy for making cost-sensitive three-way decisions. Sosnowski and Ślęzak [18] introduce a model of networks of comparators for solving problems of object identification, in which a sequence of comparators is used for decision-making. In this paper, I present another way to formally formulate sequential three-way decisions through the notion of multiple levels of granularity. The main components of the proposed model are discussed below.

Multiple levels of granularity. We assume that there are $n+1$, $n \geq 1$, levels of granularity. For simplicity, we use the index set $\{0, 1, 2, \ldots, n\}$ to denote the $n+1$ levels, with 0 representing the finest granularity (i.e., the ground level) and n the coarsest granularity. The simple two-way decisions can be viewed as decision-making at the ground level 0. For sequential three-way decisions, we assume that a three-way decision is made at levels $n, n-1, \ldots, 1$ and a two-way decision is made at the ground level 0. That is, the final result of sequential three-way decisions is a two-way decision. At each stage, only objects with a non-commitment decision will be further explored in the next level.

Multiple descriptions of objects. With $n+1$ levels, we have $n+1$ distinct representations and descriptions of the same object at different levels. Suppose

$$\mathrm{Des}_0(x) \preceq \mathrm{Des}_1(x) \preceq \ldots \preceq \mathrm{Des}_n(x), \tag{4}$$

is a sequence of descriptions of object $x \in U$ with respect to $n+1$ levels of granularity. The relation $\preceq$ denotes a "finer than" relationship between different descriptions. A description at a coarser level is more abstract by removing some details of description in a finer level. It may be commented that the languages used to describe objects may be different at different levels. Consequently, the processing methods and costs may also be different.

Multiple evaluations of objects. Due to different representations at different levels, we need to consider different evaluations too. Let v_i, $0 \leq i \leq n$, denote an evaluation at level i whose values are from a totally ordered sets $(L_i, \preceq_i)$. In contrast to the strategy of simple two-way decision making, in a sequential three-way decision process the same object may be evaluated in several levels. Therefore, we must consider the extra costs of the decision process at different levels. The costs may include, for example, the cost needed for obtaining new information and the cost of computing the evaluation v_i.

Three-way decisions at a particular level. Except the ground level 0, we may make three-way decisions for objects with a non-commitment decision. Suppose U_{i+1} is the set of objects with a non-commitment decision from level $i+1$. For level n, we use the entire set U as the set of objects with a non-commitment decision, i.e., $U_{n+1} = U$. For level i, $1 \leq i \leq n$, we can choose a pair of thresholds $\alpha_i, \beta_i \in L_i$ with $\beta_i \prec_i \alpha_i$. Three-way decision making can be expressed as:

$$\begin{aligned}
\mathrm{POS}_{(\alpha_i,\beta_i)}(v_i) &= \{x \in U_{i+1} \mid v_i(\mathrm{Des}_i(x)) \succeq_i \alpha_i\},\\
\mathrm{NEG}_{(\alpha_i,\beta_i)}(v_i) &= \{x \in U_{i+1} \mid v_i(\mathrm{Des}_i(x)) \preceq_i \beta_i\},\\
\mathrm{BND}_{(\alpha_i,\beta_i)}(v_i) &= \{x \in U_{i+1} \mid \beta_i \prec_i v_i(\mathrm{Des}_i(x)) \prec_i \alpha_i\}.
\end{aligned} \tag{5}$$

The boundary region gives the set of objects with a non-commitment decision, namely, $U_i = \mathrm{BND}_{(\alpha_i,\beta_i)}(v_i)$. For level 0, a two-way decision is made for the set of objects U_1 based on a single threshold $\gamma_0 \in L_0$.

Due to a lack of detailed information, one may prefer to a deferment decision to increase the chance of making a correct acceptance or rejection decision when

Algorithm 1: S3D (Sequential three-way decisions)

Input: A set of objects U, a family of descriptions for each object $\{\mathrm{Des}_i(x)\}$, a set of evaluations $\{v_i\}$, and a set of pairs of thresholds $\{(\alpha_i, \beta_i)\}$;
Output: Two regions POS and NEG;
begin
  POS $= \emptyset$;
  NEG $= \emptyset$;
  $i = n$;
  $U_{n+1} = U$;
  $U_1 = \emptyset$;
  while $U_{i+1} \neq \emptyset$ and $i > 0$ **do**
    $\mathrm{POS}_{(\alpha_i,\beta_i)}(v_i) = \{x \in U_{i+1} \mid v_i(\mathrm{Des}_i(x)) \succeq_i \alpha_i\}$;
    $\mathrm{NEG}_{(\alpha_i,\beta_i)}(v_i) = \{x \in U_{i+1} \mid v_i(\mathrm{Des}_i(x)) \preceq_i \beta_i\}$;
    $\mathrm{BND}_{(\alpha_i,\beta_i)}(v_i) = \{x \in U_{i+1} \mid \beta_i \prec_i v_i(\mathrm{Des}_i(x)) \prec_i \alpha_i\}$;
    $\mathrm{POS} = \mathrm{POS} \cup \mathrm{POS}_{(\alpha_i,\beta_i)}(v_i)$;
    $\mathrm{NEG} = \mathrm{NEG} \cup \mathrm{NEG}_{(\alpha_i,\beta_i)}(v_i)$;
    $U_i = \mathrm{BND}_{(\alpha_i,\beta_i)}(v_i)$;
    $i = i - 1$;
  if $U_1 \neq \emptyset$ **then**
    $\mathrm{POS}_{\gamma_0}(v_0) = \{x \in U \mid v_0(\mathrm{Des}_0(x)) \succeq \gamma_0\}$;
    $\mathrm{NEG}_{\gamma_0}(v_0) = \{x \in U \mid v_0(\mathrm{Des}_0(x)) \prec \gamma_0\}$;
    $\mathrm{POS} = \mathrm{POS} \cup \mathrm{POS}_{\gamma_0}(v_0)$;
    $\mathrm{NEG} = \mathrm{NEG} \cup \mathrm{NEG}_{\gamma_0}(v_0)$;
  return POS, NEG;

Fig. 1. Algorithm of sequential three-way decisions

more evidence and details are available at lower levels. This can be controlled by setting proper thresholds at different levels. Typically, one may use a larger threshold α and a smaller threshold β at a higher level of granularity [30].

By summarizing the discussion, Figure 1 gives the algorithm S3D of sequential three-way decisions. In the algorithm, the set U_1 is initialized to the empty set. It will remind to be empty if an empty boundary region is obtained before reaching the ground level 0. In addition to the construction of the evaluation and thresholds at each level, for sequential three-way decisions, one must consider the construction and interpretation of a sequence of multiple levels of granularity.

4 Comparison of Simple Two-way Decisions and Sequential Three-way Decisions

In this section, I provide an analysis of costs associated with two-way and sequential three-way decisions to demonstrate that there may be advantages to using a sequence of three-way decisions.

4.1 Total Cost of Decisions

Simple two-way decisions and sequential three-way decisions can be compared from two aspects. One is quality of the decision result in terms of errors or costs caused by incorrect decisions and the other is cost of the decisions process for arriving at a decision. Both types of cost have been well studied and widely used in comparing different algorithms of two-way classification. In comparison, the latter has received less attention, except for the case of decision-tree based classification methods [10–13, 19]. When classifying an object with a decision tree, it is necessary to perform a sequence of tests of some internal nodes of the tree. The cost of the decision process can be viewed as the total cost of all required tests. The proposed sequential three-way decisions share some similarities with decision-tree based methods, but focus more on multiple levels of granularity and multiple representations of an objects. The cost of decision process becomes an important factor [6, 8, 9].

Suppose $COST_R$ and $COST_P$ denote, respectively, the cost of the decision result and the cost of the decision process. It is reasonable to assume that the total cost of decisions is a function for pooling together the two costs, that is,

$$COST = F(COST_R, COST_P). \tag{6}$$

There are many choices of the function F. Two special forms of the function are the simple linear combination and product:

$$\begin{aligned} COST' &= w_R * COST_R + w_P * COST_P, \\ COST'' &= (COST_R)^a * (COST_P)^b, \end{aligned} \tag{7}$$

where the weights $w_R \geq 0$, $w_P \geq 0$ and $w_R + w_P \neq 0$, and $a \geq 0$, $b \geq 0$ and $a + b \neq 0$, represent respectively the relative importance of the two types of costs. There seems to be an inverse relationship between the two types of costs. A decision-making method may produce a high quality result but tends to require a large processing cost. It may also happen that a decision-making method may require a small processing cost but produces a low quality result. In general, there is a trade-off between the two types of costs. Finding the right balance holds the key to making effective decisions.

4.2 Cost of the Decision Result

The result of simple two-way decisions and the final result of sequential three-way decisions are, respectively, a division of U into two regions POS and NEG. Some of the decisions of acceptance and rejection for constructing the two regions may, in fact, be incorrect. Let $S_1 = \{x \in U \mid s(x) = 1\}$ be the set of objects in state 1 and $S_0 = \{x \in U \mid s(x) = 0\}$ be the set of objects in state 0. Table 1 summarizes the errors and costs of various decisions, where $S = 1$ and $S = 0$ denote the two states of objects and $|\cdot|$ denotes the cardinality of a set.

Table 1. Information of decision result

	$s(x)=1$ (P)	$s(x)=0$ (N)	total
a_A: accept	Correct acceptance $\lvert \mathrm{POS} \cap S_1 \rvert$	Incorrect acceptance $\lvert \mathrm{POS} \cap S_0 \rvert$	$\lvert \mathrm{POS} \rvert$
a_R: reject	Incorrect rejection $\lvert \mathrm{NEG} \cap S_1 \rvert$	Correct rejection $\lvert \mathrm{NEG} \cap S_0 \rvert$	$\lvert \mathrm{NEG} \rvert$
total	$\lvert S_1 \rvert$	$\lvert S_0 \rvert$	$\lvert U \rvert$

(a) Errors of decision result

	$s(x)=1$ (P)	$s(x)=0$ (N)
a_A: accept	$\lambda_{AP}=\lambda(a_A \lvert S=1)$	$\lambda_{AN}=\lambda(a_A \lvert S=0)$
a_R: reject	$\lambda_{RP}=\lambda(a_R \lvert S=1)$	$\lambda_{RN}=\lambda(a_R \lvert S=0)$

(b) Costs of decision result

The rates of two types of error, i.e., incorrect acceptance error (IAE) and incorrect rejection error (IRE), are given by:

$$\begin{aligned} IAE &= \frac{|\mathrm{POS} \cap S_0|}{|\mathrm{POS}|}, \\ IRE &= \frac{|\mathrm{NEG} \cap S_1|}{|\mathrm{NEG}|}, \end{aligned} \tag{8}$$

where we assume that the positive and negative regions are nonempty, otherwise, the corresponding rate of error is defined as 0. Let $a(x)$ denote a decision made for object x. The total cost of decision results of all objects is computed as,

$$\begin{aligned} COST_R &= \sum_{x \in U} \lambda(a(x)|S = s(x)) \\ &= |\mathrm{POS} \cap S_1| * \lambda(a_A|S=1) + |\mathrm{POS} \cap S_0| * \lambda(a_A|S=0) + \\ &\quad |\mathrm{NEG} \cap S_1| * \lambda(a_R|S=1) + |\mathrm{NEG} \cap S_0| * \lambda(a_R|S=0) \\ &= |\mathrm{POS}| * ((1 - IAE) * \lambda(a_A|S=1) + IAE * \lambda(a_A|S=0)) + \\ &\quad |\mathrm{NEG}| * (IRE * \lambda(a_R|S=1) + (1 - IRE) * \lambda(a_R|S=0)). \end{aligned} \tag{9}$$

The total cost of decision result is related to the two types of decision error. The rates of errors and total cost may be used to design an objective function for finding an optimal threshold γ in simple two-way decisions.

Consider a special cost function defined by:

$$\begin{aligned} \lambda_{AP} &= 0, \qquad \lambda_{AN} = 1; \\ \lambda_{RP} &= 1, \qquad \lambda_{RN} = 0. \end{aligned} \tag{10}$$

There is a unit cost for an incorrect decision and zero cost for a correct decision. By inserting this cost function in to Equation (9), we have

$$\begin{aligned} COST_R &= |\mathrm{POS}| * IAE + |\mathrm{NEG}| * IRE \\ &= |\mathrm{POS} \cap S_0| + |\mathrm{NEG} \cap S_1|. \end{aligned} \tag{11}$$

The first expression suggests that the cost is a weighted sum of the two rates of incorrect decisions. The cost based measure is more informative than rates of incorrect decision, as the latter can be viewed as a special case of the former. The second expression suggests that the cost is the number of objects with an incorrect decision.

4.3 Costs of the Decision Process

For simple two-way decisions, we assume that all decisions are made at the ground level 0. The cost for processing each object is C_0 and the cost of the decision process is given by:

$$COST_{2P} = |U| * C_0. \tag{12}$$

When the cost C_0 is very large, the cost of the decision process $COST_{2P}$ may be very high. For many decision-making problem, we may not need to acquire all information of the ground level 0. This suggests a strategy of sequential decisions in which additional information is gradually acquired when it is necessary.

Let C_i denote the cost needed for evaluating an object at level i. It is reasonable to assume,

$$C_0 > C_i > 0, \quad i = n, n-1, \ldots, 1. \tag{13}$$

That is, the cost of the decision process at an abstract level is strictly less than at the ground levels; otherwise, we will not have any advantages of using the strategy of sequential three-way decision making. The magnitudes of C_i's depend on special applications. Consider a special case where C_i represents time needed for computing the evaluation at level i. We can assume that

$$C_n < C_{n-1} < \ldots < C_0. \tag{14}$$

This is equivalent to saying that we can make a faster decision at a higher level of granularity, as we do not have to consider minute details of the lower levels.

According to the condition $C_0 > C_i > 0, \quad i = n, n-1, \ldots, 1$, if we can make a definite decision of an acceptance or a rejection at higher levels of granularity, we may be able to avoid a higher cost at the ground level 0. Let $l(x)$ denote the level at which a decision of an acceptance or a rejection is made for x. The object x is considered in all levels from level n down to level $l(x)$. The processing cost of x can be computed as:

$$COST_{3P}(x) = \sum_{i=l(x)}^{n} C_i = C_{n \to l(x)}, \tag{15}$$

where $C_{n\to i}$ denote the cost incurred from level n down to level i. The total processing cost for all objects can be computed as follows:

$$\begin{aligned} COST_{3P} &= \sum_{x\in U} COST_{3P}(x) \\ &= \sum_{i=0}^{n} (|\mathrm{POS}_{(\alpha_i,\beta_i)}(v_i)| + |\mathrm{NEG}_{(\alpha_i,\beta_i)}(v_i)|) * C_{n\to i}. \end{aligned} \tag{16}$$

According to this equation, if the cost C_0 is very large and we can make an acceptance or a rejection decision for a majority of objects before reaching the ground level 0, the advantages of sequential three-way decisions will be more pronounced. On the other hand, if a definite decision of an acceptance or a rejection is made for the majority of objects at lower levels of granularity, sequential three-way decisions would be inferior.

To gain more insights into sequential three-way decisions, let us consider a special composition of the cost C_i:

$$C_i = C_i^E + C_i^A, \tag{17}$$

where C_i^E denotes the cost for computing the evaluation v_i and C_i^A denotes the cost for acquiring additional information at level i. For this interpretation, we have the following assumption:

$$C_n^E \leq C_{n-1}^E \leq \ldots \leq C_0^E.$$

The assumption suggests that the cost for computing the evaluation function is lower at a higher level granularity due to the omission of detailed information. For simple two-way decisions at ground level 0, we must consider all information acquired from levels n down to 1. For an object x, the costs of decision processes of simple two-way decisions and sequential three-way decisions are given, respectively, by:

$$\begin{aligned} COST_{2P}(x) &= C_0^E + C_{n\to 0}^A, \\ COST_{3P}(x) &= C_{n\to l(x)}^E + C_{n\to l(x)}^A. \end{aligned} \tag{18}$$

It follows that

$$COST_{2P}(x) - COST_{3P}(x) = C_{(l(x)-1)\to 0}^A - (C_{n\to l(x)}^E - C_0^E). \tag{19}$$

The first term represents the extra cost of simple two-way decisions for acquiring extra information from level $l(x)-1$ down to level 0, and the second term represents the extra cost of sequential three-way decisions in computing evaluations from level n down to level $l(x)$. That is, sequential three-way decisions reduce the cost of acquiring information at the expense of computing additional evaluations. If the difference in Equation (19) is greater than 0, then sequential three-way decisions have an advantage of a lower cost of the decision process. In situations where the cost of acquiring new information is more than the cost of computing evaluations, sequential three-way decisions are superior to simple two-way decisions at the ground level 0 with respect to the cost of decision process. In addition, when simple two-way decisions and sequential three-way decisions produce decision results of comparable quality, sequential three-way decisions are a better choice.

In general, we want to have sequential three-way decisions that produce the similar decision quality as simple two-way decisions but have a lower cost of decision process. To achieve this goal, one needs study carefully the cost structures of sequential three-way decisions in order to determine the best number of levels and best thresholds at each level. This implies that designing a sequential three-way decision procedure is more difficulty than designing a simple two-way decision procedure. There are many challenging problems to be solved for sequential three-way decisions.

5 Conclusion

In this paper, I present a granular computing perspective on sequential three-way decisions. Multiple levels of granularity lead to multiple representations of the same object, which in turn leads to sequential three-way decisions. Sequential decisions rely on a basic principle of granular computing, i.e., one only examines lower levels of granularity if there is a benefit. By considering the cost of the decision process, I show that a sequential three-way decision strategy may have a lower cost of the decision process than a simple two-way decision strategy, as the former may require less information and demand less time for computing evaluations at higher levels of granularity. Sequential three-way decisions are particularly useful for practical decision-making problems when information is unavailable and is acquired on demands with associated cost.

Sequential three-way decisions are much more complicated than simple two-way decisions. There are many challenging issues. One must construct multiple levels of granularity and multiple representations of the same object. One must consider more parameters, such as the number of levels, evaluations at different levels, and the thresholds at each level. One must also study cost structures that make sequential three-way decisions a better strategy.

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